

# D-brane Instantons in Supersymmetric 4D String Vacua

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In all classes one can build semirealistic models, which come with

- MSSM like physics
- many unobserved particles, in particular singlet states (moduli)

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(Most of the work so far was for world-sheet instantons in Type II and heterotic string theory and for M-brane instantons)

(Dine, Seiberg, Wen, Witten), (Becker<sup>2</sup>, Strominger), (Harvey, Moore), (Witten), (Green, Gutperle), (Antoniadis, Gava, Narain, Taylor), (Rocek, Saueressig, Theis, Vandoren), (Berglund, Mayr), (Kashani-Poor, Tomasiello), (Tsimpis), (Halmagyi, Melnikov, Sethi), (Grimm) ...

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  - Stringy derivation of field theory instanton effects

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$$\prod_a U(N_a) = \prod_a SU(N_a) \times U(1)_a$$

in general contains **anomalous**  $U(1)_a$  symmetries



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- Only specific linear combinations of  $U(1)$ s are **massless** and remain as unbroken gauge symmetry (like  $U(1)_Y$ )
- Global  $U(1)$  **forbid** some desirable matter **couplings**, e.g. Majorana type **neutrino masses**,  $SU(5)$  Yukawa couplings or  $\mu$ -terms  $\rightarrow$  relation to M-theory on  $G_2$  manifolds(?)

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Consider: D2-brane (E2) instantons in **Type IIA** wrapping a sLag three-cycle  $\Xi$  on Calabi-Yau.

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Consider: D2-brane (E2) instantons in **Type IIA** wrapping a sLag three-cycle  $\Xi$  on Calabi-Yau.

From E2-E2 open strings:

- Generic 4 **bosonic** zero modes  $X_\mu$  and 4 **fermionic** zero modes  $\theta^\alpha$  and  $\bar{\theta}^{\dot{\alpha}}$
- Due to deformations,  $b_1(\Xi)$  complex bosonic zero modes  $Y_i$  and fermionic zero modes  $\mu_i^\alpha$  and  $\bar{\mu}_i^{\dot{\alpha}}$



# F-terms via E2-Instantons

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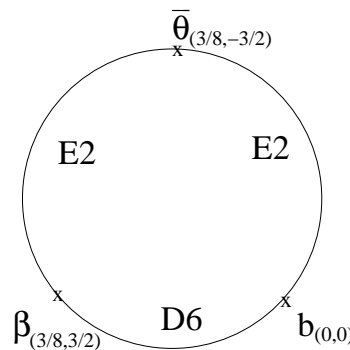
F-terms possible only if

- The two  $\bar{\theta}^{\dot{\alpha}}$  zero modes are projected out by  $\Omega\bar{\sigma}$ . For this the E2 must be invariant under  $\bar{\sigma}$  and must be an  $O(1)$  instanton (instead of  $SP(2)$  or  $U(1)$ ) (Argurio, Bertolini, Ferreti, Lerda, Petersson) , (Ibanez, Schellekens, Uranga) , (Bianchi, Fucito, Morales)

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- The two  $\bar{\theta}^{\dot{\alpha}}$  zero modes can be absorbed elsewhere, like for instantons on top of D6-brane:



→ fermionic ADHM-constraints (Billo et al., hep-th/0211250) ,

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E2-E2' instanton recombination:

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- $[E2 \cap E2']^\pm = 1$ : After recombination  $\bar{\theta}$  are soaked up and  $m, \bar{\mu}_{\dot{\alpha}}$  zero modes survive (deformations of the instantons)  $\rightarrow$  generation of Beasley/Witten type multi-fermion couplings (Beasley, Witten)

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- In Type IIB  $\Omega I_6(-1)^{F_L}$  orientifolds a **primitive**  $G_{2,1}$  flux does **not** lift the  $\bar{\theta}$  zero modes of an U(1) instanton

# Type II Space-time Instantons

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Instanton action:

$$W_{np} \propto e^{-S_{E2}} = \exp \left[ -\frac{2\pi}{\ell_s^3} \left( \frac{1}{g_s} \int_{\Xi} \Re(\Omega_3) - i \int_{\Xi} C_3 \right) \right]$$

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Indeed

$$e^{-S_{E2}} \rightarrow e^{i Q_a(E2) \Lambda_a} e^{-S_{E2}},$$

where

$$Q_a(E2) = N_a \Xi \circ (\Pi_a - \Pi'_a).$$

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Consequence: If  $Q_a(E2) \neq 0$  for some  $a$ , no terms

$W = e^{-S_{E2}}$  possible but:

$$W = \prod_i \Phi_i e^{-S_{E2}} \quad \text{with} \quad \sum_i Q_a(\Phi_i) + Q_a(E2) = 0 \quad \forall a$$

i.e. **non-perturbative** breakdown of global  $U(1)$  symmetries.

see also e.g. : (Achúcarro, Carlos, Casas, Doplicher, hep-th/0601190), (Haack, Krefl, Lüst, Van Proeyen, Zagermann, hep-th/0609211)

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How can we understand this selection rule in terms of **fermionic** zero modes?

# Instanton zero modes



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Additional Zero modes charged under  $U(1)_a$ :

Strings between  $E2$  and  $D6_a$  have **DN**-boundary conditions in 4D and mixed boundary conditions along  $CY_3 \rightarrow$

**1/2 complex fermionic** zero mode  $\lambda_a$  (Ganor, hep-th/9612077)

zero modes	Reps.	number
$\lambda_{a,I}$	$(-1_E, \square_a)$	$I = 1, \dots, [\Xi \cap \Pi_a]^+$
$\bar{\lambda}_{a,I}$	$(1_E, \bar{\square}_a)$	$I = 1, \dots, [\Xi \cap \Pi_a]^-$
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Total  $U(1)_a$  **charge** of all zero modes:

$$Q_a(E2) = N_a \Xi \circ (\Pi_a - \Pi'_a).$$

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E2-instantons are described by open strings  $\rightarrow$  computation of stringy instanton correlation functions should be possible in (boundary) conformal field theory. (Gutperle, Green, hep-th/9701093), (Billo et al., hep-th/0211250)

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As a first step we would like to compute (rigid) E2-contributions to the charged matter field superpotential

$$W_{np} \simeq \prod_{i=1}^M \Phi_{a_i, b_i} e^{-S_{E2}}.$$

with  $\Phi_{a_i, b_i} = \phi_{a_i, b_i} + \theta\psi_{a_i, b_i}$  denoting chiral matter superfields at the intersection of  $\Pi_{a_i}$  with  $\Pi_{b_i}$  (suppress Chan-Paton labels for simplicity).

# Instanton calculus: Summary

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Probe superpotential by correlator

$$\langle \Phi_{a_1, b_1} \cdot \dots \cdot \Phi_{a_M, b_M} \rangle_{E2\text{-inst}} = \frac{e^{\frac{\kappa}{2}} Y_{\Phi_{a_1, b_1}, \dots, \Phi_{a_M, b_M}}}{\sqrt{K_{a_1, b_1} \cdot \dots \cdot K_{a_M, b_M}}}$$

$$\begin{aligned} & \langle \Phi_{a_1, b_1}(x_1) \cdot \dots \cdot \Phi_{a_M, b_M}(x_M) \rangle_{E2\text{-inst}} = \\ & = \int d^4x d^2\theta \sum_{\text{conf.}} \prod_a \left( \prod_{i=1}^{[\Xi \cap \Pi_a]^+} d\lambda_a^i \right) \left( \prod_{i=1}^{[\Xi \cap \Pi_a]^-} d\bar{\lambda}_a^i \right) \\ & \quad \exp(-S_{E2}) \times \exp(Z'_0) \\ & \quad \times \langle \widehat{\Phi}_{a_1, b_1}[\vec{x}_1] \rangle_{\lambda_{a_1}, \bar{\lambda}_{b_1}}^{\text{tree}} \cdot \dots \cdot \langle \widehat{\Phi}_{a_L, b_L}[\vec{x}_L] \rangle_{\lambda_{a_L}, \bar{\lambda}_{b_L}}^{\text{tree}} \times \\ & \quad \prod_k \langle \widehat{\Phi}_{c_k, c_k}[\vec{x}_k] \rangle_{A(E2, D\delta_{c_k})}^{\text{loop}} \end{aligned}$$

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- Factor off **vacuum loops** involving at least one  $E2$  boundary:

$$Z^A(E2, D6_a) = c \int_0^\infty \frac{dt}{t} \text{Tr}_{E2, D6_a} \left( e^{-2\pi t L_0} \right) \neq 0$$

and likewise  $Z^M(E2, O6) \neq 0$  but  $Z^A(E2, E2) = 0$  (due to bose-fermi deg.).

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Therefore

$$\exp(Z_0) = \exp \left( \sum_a Z^A(E2, D6_a) + Z^M(E2, O6) \right)$$

**One-loop determinants!**

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Diagrammatically we have the **relation** (for even spin structures)

The diagram shows an equality between two cylinder-like structures. On the left, a cylinder has two vertical ovals representing cross-sections, labeled  $E2_a$  on the left and  $D_b$  on the right. On the right, an equals sign is followed by another cylinder with two vertical ovals. The left oval is labeled  $D_a$  and has  $F_a \times$  written above it and  $\times F_a$  written below it. The right oval is labeled  $D_b$ .

(Abel, Goodsell), (Akerblom, Bl, Lüst, Plaushinn, Schmidt-Sommerfeld)

Open problem: Computation of **odd** spin-structure  $E2 - D6$  amplitude.

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Stringy **one-loop** amplitudes are known to include the **holomorphic Wilsonian** part and **non-holo.** contributions from wave-function normalisation

(Shifman, Vainshtein), (Kaplunovsky, Louis)

$$\begin{aligned} Z_0(E2_a) = & -\text{Re}(f_W^a)_{1\text{-loop}} - \frac{b_a}{2} \ln \left[ \frac{M_p^2}{\mu^2} \right] - \frac{c_a}{2} \mathcal{K}_{\text{tree}} \\ & - \ln \left( \frac{V_3}{g_s} \right)_{\text{tree}} + \sum_b \frac{|I_{ab} N_b|}{2} \ln [\det Z_{(r)}]_{\text{tree}} \end{aligned}$$

with

$$b_a = \sum_b \frac{|I_{ab} N_b|}{2} - 3, \quad c_a = \sum_b \frac{|I_{ab} N_b|}{2} - 1.$$

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The CFT disc amplitudes combine non-holomorphic and holomorphic pieces

$$\langle \widehat{\Phi}_{a,b}[\vec{x}] \rangle_{\lambda_a, \bar{\lambda}_b} = \frac{e^{\frac{\kappa}{2}} Y_{\lambda_a} \widehat{\Phi}_{a,b}[x] \bar{\lambda}_b}{\sqrt{K_{\lambda_a, a} \widehat{K}_{a,b}[x] K_{b, \bar{\lambda}_b}}} .$$

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Therefore, all the non-holomorphic piece including the instanton cancel out and one gets the **holomorphic** quantity

$$Y_{\Phi_{a_1, b_1}, \dots, \Phi_{a_M, b_M}} = \sum_{\text{conf.}} \exp(-S_{E2})_{\text{tree}} \exp(-f_W^a)_{1\text{-loop}} \\ Y_{\lambda_{a_1}} \widehat{\Phi}_{a_1, b_1}[\vec{x}_1] \bar{\lambda}_{b_1} \cdot \dots \cdot Y_{\lambda_{a_L}} \widehat{\Phi}_{a_L, b_L}[\vec{x}_L] \bar{\lambda}_{b_L}.$$

**Higher loop** only contribute to corrections of Kähler potentials.

# Applications : Moduli potential

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For E2-instantons with **no matter field zero modes** corrections to the uncharged closed/open string moduli **superpotential** can be generated

$$W = A(T, \Delta) e^{-U}$$

- Vacuum destabilisation
- KKLT like stabilisation of closed string moduli
- Inflaton potential for D-brane modulus  $\Delta$  (Baumann et. al. [hep-th/0607050](https://arxiv.org/abs/hep-th/0607050))

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Non-pert. **Majorana** coupling:

$$W_M = M_M (N_R)^c (N_R)^c$$

with

$$M_M = x M_s e^{-\frac{2\pi}{\ell_s^3 g_s} \text{Vol}_{E2}}$$



# Applications : matter couplings

For appropriate E2-instantons, important perturbatively excluded **matter couplings** can be generated

- **Majorana masses** for right-handed neutrinos (Bl, Cvetič, Weigand, hep-th/0609191), (Ibanez, Uranga, hep-th/0609213), see also (Bianchi, Kiritsis), (Cvetič, Richter, Weigand), (Ibanez, Schellekens, Uranga), (Antusch, Ibanez, Macri)

Non-pert. **Majorana** coupling:

$$W_M = M_M (N_R)^c (N_R)^c$$

with

$$M_M = x M_s e^{-\frac{2\pi}{\ell_s^3 g_s} \text{Vol}_{E2}}$$

The **natural mass scale** is  $M_s \simeq M_{\text{GUT}}$  so that  $M_M$  is non-pert. suppressed w.r.t. to  $M_s \gg M_{\text{weak}}$ !

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Consider  $SU(5)$  GUT model via intersecting D6-branes.

sector	number	$U(5)_a \times U(1)_b$ reps.	$U(1)_X$
$(a', a)$	3	$\mathbf{10}_{(2,0)}$	$\frac{1}{2}$
$(a, b)$	3	$\bar{\mathbf{5}}_{(-1,1)}$	$-\frac{3}{2}$
$(b', b)$	3	$\mathbf{1}_{(0,-2)}$	$\frac{5}{2}$
$(a', b)$	1	$\mathbf{5}_{(1,1)}^H + \bar{\mathbf{5}}_{(-1,-1)}^H$	$(-1) + (1)$

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Perturbative Yukawa couplings

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Yukawa coupling

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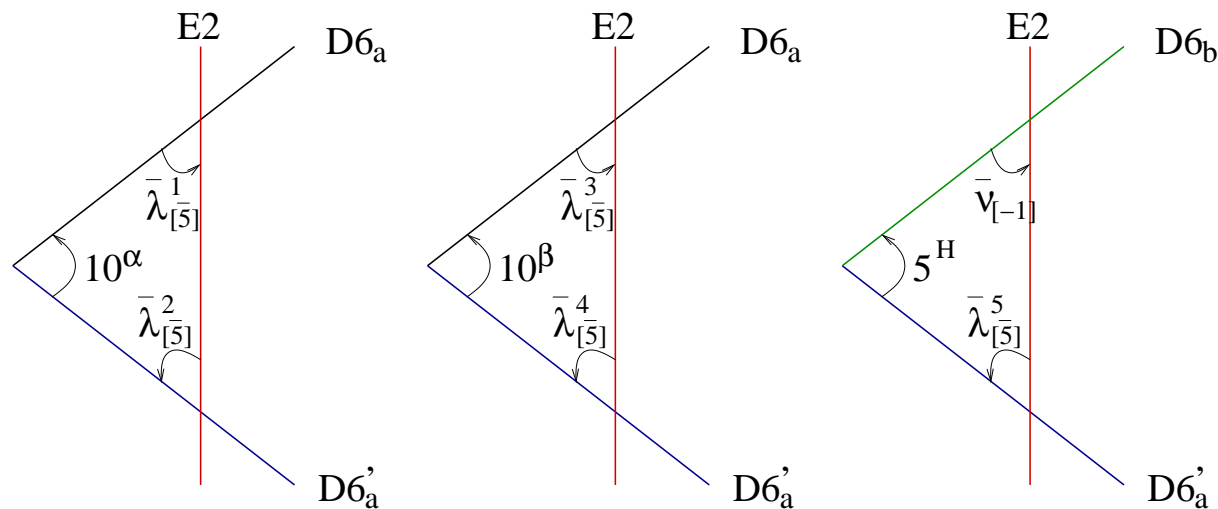
is not  $U(1)$  invariant (but present on  $G_2$  manifolds).

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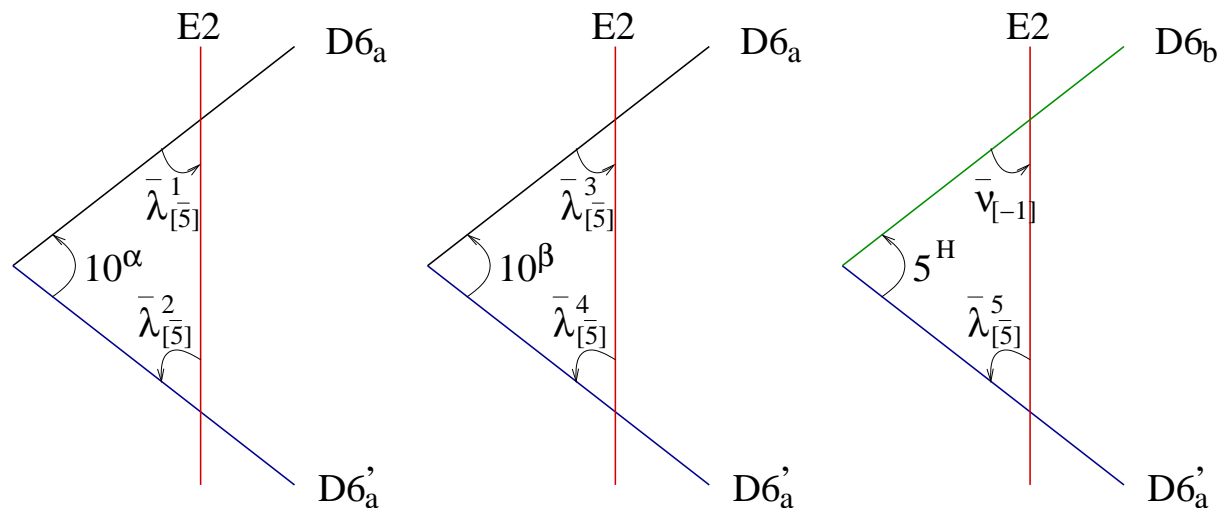
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$$W_Y = Y_{\langle \mathbf{10} \mathbf{10} \mathbf{5}_H \rangle}^{\alpha\beta} \epsilon_{ijklm} \mathbf{10}_{ij}^\alpha \mathbf{10}_{kl}^\beta \mathbf{5}_m^H$$

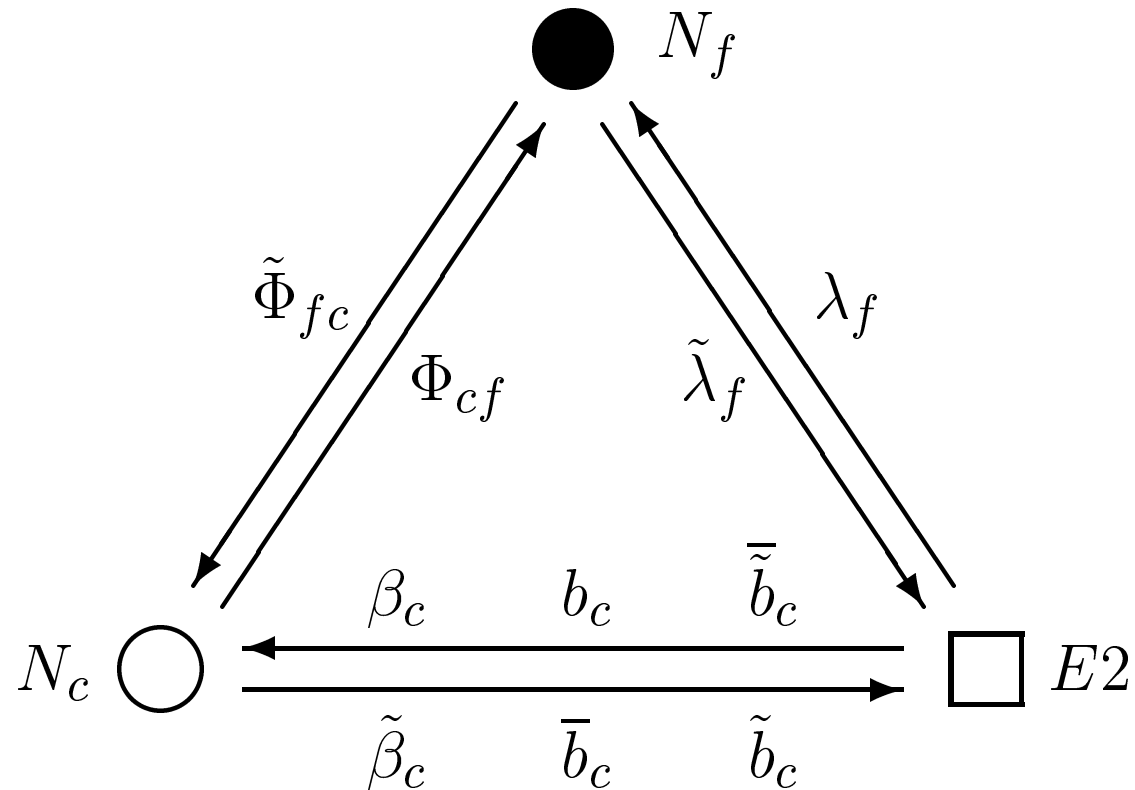
Flipped  $SU(5)$ : hierarchy between  $(d, s, b)$  and  $(u, c, t)$  by E2-instanton, flavour hierarchy by world-sheet instantons



# Applications: The ADS superpotential

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$N=1$  SQCD with  $N_f = N_c - 1$  flavours



(Akerblom, Bl, Lüst, Plauschinn, Schmidt-Sommerfeld, hep-th/0612132)

(Florea, Kachru, McGreevy, Saulina, hep-th/0610003)

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Eventually one arrives at

$$S_W \simeq \int d^4x d^2\theta \frac{\Lambda^{3N_c - N_f}}{\det[M_{ff'}]}.$$

generalisations (Argurio, Bertolini, Ferreti, Lerda, Petersson), (Bianchi, Fucito, Morales),

(Billo, Frau, Pesando, Di Vecchia, Lerda, Marotta)

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Holomorphy dictates that for D6-branes the **holomorphic gauge kinetic function** must look like

$$f = \sum_I M_a^I U_I^c + f^{1\text{-loop}} \left( e^{-T_i^c} \right) + f^{\text{np}} \left( e^{-U_I^c}, e^{-T_i^c} \right).$$

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For intersecting **D6-branes on  $T^6$**  the holomorphic **one-loop gauge threshold corrections** are: (Lüst, Stieberger) , (Akerblom, Bl, Lüst, Schmidt-Sommerfeld )

- $\mathcal{N} = 1$  sector:  $f^{(1)} = 0$
- $\mathcal{N} = 2$  sector:  $f^{(1)} = \ln(\eta(i T^c))$

**World-sheet instanton** corrections come from world-sheets with **two** boundaries  $\rightarrow$  expect **E2-instantons** from **non-rigid** ones with  $b_1(\Xi) = 1$ .

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Zero modes:  $Y_i, \mu^\alpha, \bar{\mu}^{\dot{\alpha}}$ . Distinguish **two cases** depending on how the **anti-holomorphic involution**  $\bar{\sigma}$  acts on the open string modulus  $Y$

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The zero mode **measure** reads

$$\int d^4x d^2\theta d^2y d^2\bar{\mu} e^{-S_{E2}} \dots, \quad \text{for } \bar{\sigma} : y \rightarrow y$$

and

$$\int d^4x d^2\theta d^2\mu e^{-S_{E2}} \dots, \quad \text{for } \bar{\sigma} : y \rightarrow -y.$$

(dual to world-sheet instantons studied by Beasley-Witten)

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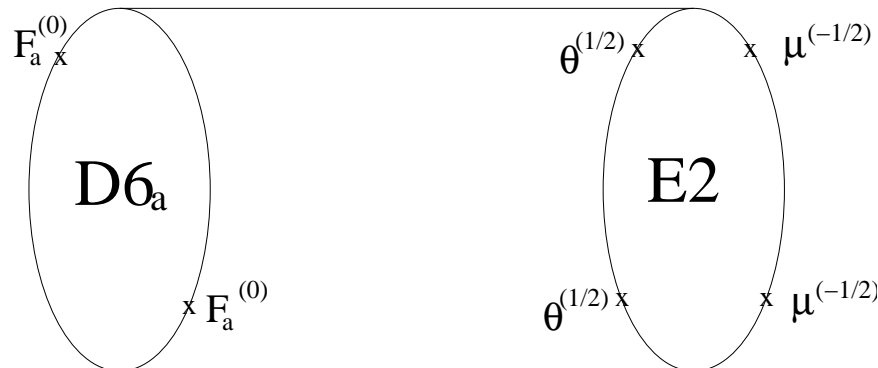
$$\langle F_a(p_1) F_a(p_2) \rangle_{E2} = \int d^4x d^2\theta d^2\mu \exp(-S_{E2}) \exp(Z'_0(E2)) A_{F_a^2}(E2, D6_a)$$

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$$\xi_a = \int_{\Pi_a} \mathfrak{F}(\Omega_3).$$

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Expect also **E2-brane instanton** corrections  $\rightarrow$  stability of D-branes

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