Grand Unification with & without supersymmetry

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BW2007, Kladovo, Sep. 2-9, 2007

Outline

- GUTs and neutrino mass
- Intermediate scales
- The see-saw and the Yukawa sector
- Non supersymmetric models
- The minimal SUSY model
- Departing from the minimal

GUTs and neutrino mass

SO(10): all fermions in 16 representation

SU(5) fermions: in 5 and 10 representations

$\Longrightarrow \nu_R$ is a singlet

- adding a singlet to the theory gives a lot of new parameters
- SU(5) breaks directly to SU(3)xSU(2)xU(1)

- no intermediate scales

... and m_ν calls for intermediate scales

THE (B-L) BREAKING SCALE

Best idea for small m_ν : the see-saw mechanism

give ν_R a mass by breaking B-L

at a large scale M_R

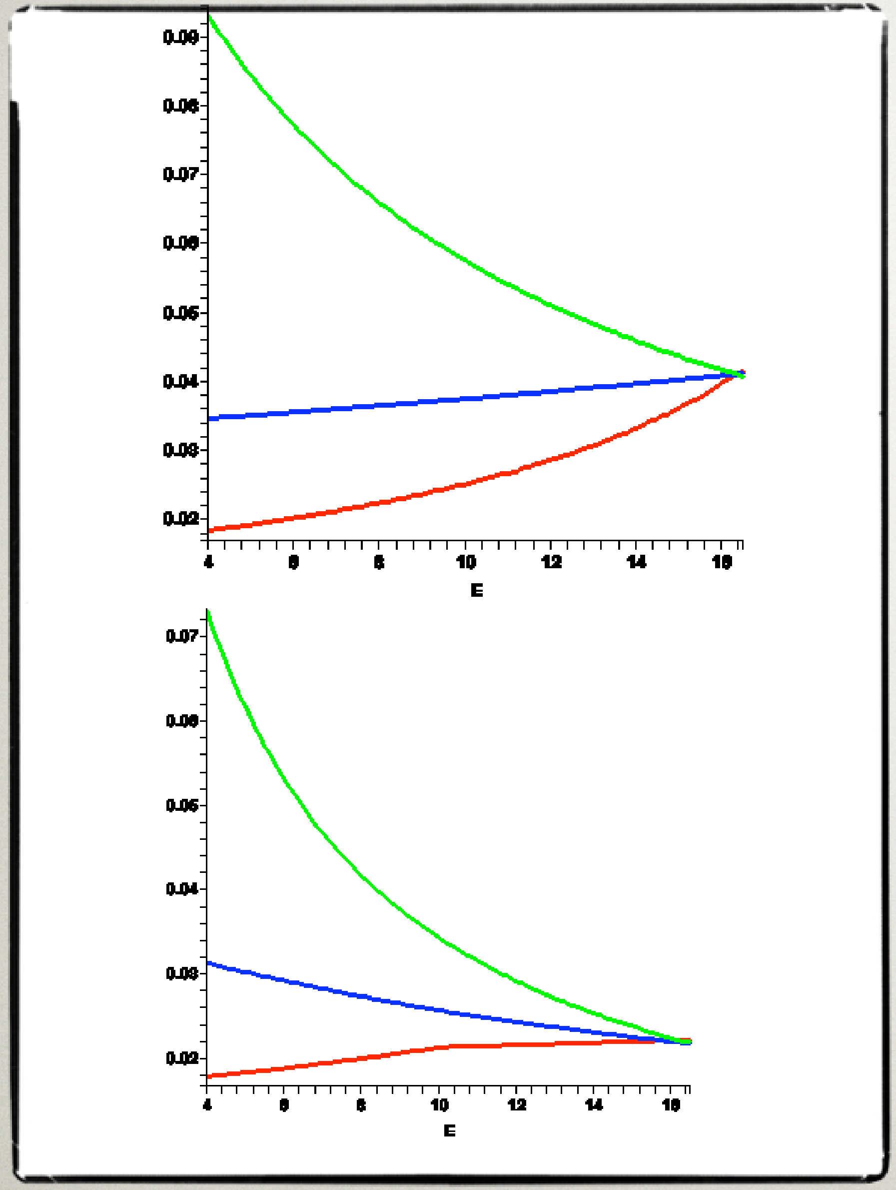
$$\langle\Delta\rangle \nu_R^T i\sigma_2 \nu_R \qquad \langle\Delta\rangle = M_R$$

$$m_\nu = \frac{M_W^2}{M_R} \qquad m_\nu \sim 0.01 eV$$

$$M_R \sim 10^{13} GeV$$

An intermediate scale would be convenient

(not indispensable)



SUSY: ONE-STEP UNIFICATION

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_U} - \frac{b_i}{2\pi}\ln(M_G/M_W)$$

$$M_G \sim 10^{16} GeV$$

NON-SUSY: INTERMEDIATE SCALE

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_U} - \frac{b_i}{2\pi}\ln(M_R/M_W) - \frac{b_i'}{2\pi}\ln(M_G/M_R)$$

M_R determined by the particle content

SO(10) symmetry breaking

Many possible intermediate scales

$$SO(10)$$

$$M_X \Downarrow \langle p \rangle$$

GUT scale

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$M_{PS} \Downarrow \langle a \rangle$$

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$M_R \Downarrow \langle \sigma \rangle$$

see-saw scale

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

TWO TYPES OF SEE-SAW

TYPE I (renormalizable version)

- An $SU(2)_R$ triplet with $(B-L)=2$ gets a vev at a large scale M_R

$$\langle \Delta^c \rangle \Rightarrow \nu^c \text{ mass} \sim M_R$$

gives a mass to the right-handed neutrino

- At EW scale, neutrino gets a Dirac mass m_D

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \quad \rightsquigarrow \quad m_\nu \sim \frac{m_D^2}{M_R} \sim \frac{M_W^2}{M_R}$$

Mohapatra, Senjanovic, 1980

Type II

In Left-Right theories, terms like:

$$\Delta\, H^2 \Delta^c \;+\; m_\Delta \Delta^2$$

H : bidoublet

Δ : Left-handed triplet

Δ^c : Right-handed triplet

Provide a small vev for the Left-handed triplet after EW breaking

$$\langle\Delta\rangle \sim \frac{\langle H\rangle^2 \langle\Delta^c\rangle}{m_\Delta^2} \sim \frac{M_W^2}{M_R}$$

Mass for ν from $L^T \tau_2 \langle\Delta\rangle L$

vev of Δ^c induces a small vev for Δ after EW breaking

In SUSY SO(10) , triplets are in 126:
mixing with 54 or 210 can give such terms in the potential.

TWO TYPES OF SEE-SAW ARE OF SAME MAGNITUDE:
BUT VERY DIFFERENT PARAMETERS INVOLVED

Yukawa sector

Pati-Salam fourth color:

$$U = \begin{pmatrix} u \\ u \\ u \\ \nu \end{pmatrix} \quad D = \begin{pmatrix} d \\ d \\ d \\ e \end{pmatrix} \ldots$$

SO(10):

$$\Psi_{16} = \begin{pmatrix} U \\ D \\ D^c \\ U^c \end{pmatrix}$$

- All fermions in one (spinorial) representation
- Couple to:

$$\Psi \, C \, \Gamma^a \Psi \; H_a \qquad \underline{10}$$

$$\Psi \, C \, \Gamma^a \Gamma^b \Gamma^c \Psi \, D_{abc} \qquad \underline{120} \ \text{(antisym.)}$$

$$\Psi \, C \, \Gamma^a \Gamma^b \Gamma^c \Gamma^d \Gamma^e \Psi \, \Sigma_{abcde} \qquad \underline{126}$$

SU(4)c x SU(2)L x SU(2)R Decomposition

$$H_{10} \;=\; (6,1,1) + (1,2,2)$$

$$D_{120} \;=\; (\overline{10},1,1) + (10,1,1) + (6,3,1) + (6,1,3) + (1,2,2) + (15,2,2)$$

$$\overline{\Sigma}_{\overline{126}} = (10,1,3) + (\overline{10},3,1) + (6,1,1) + (15,2,2)$$

$$\Delta_R \qquad\qquad \Delta_L$$

- 126 can give type I and type II see-saw
- (15,2,2) in 126 can contain the SM Higgs
  - is 126 enough for all fermion masses ? no..

One doublet is not enough:

Lazarides, Shafi Wetterich 1981

Clark, Kuo Nakagawa 1982

$$M_U = y_{10} \langle 1,2,2 \rangle^u_{10} \; + y_{126} \langle 15,2,2 \rangle^u_{126}$$

$$M_D = y_{10} \langle 1,2,2 \rangle^d_{10} \; + y_{126} \langle 15,2,2 \rangle^d_{126}$$

$$M_E = y_{10} \langle 1,2,2 \rangle^d_{10} \; - 3\, y_{126} \langle 15,2,2 \rangle^d_{126}$$

- only 10 : $m_d = m_l$
- only 126: $3\, m_d = m_l$

} at the GUT scale, for all generations

- 126 required for neutrino mass - but what else?
  - is there a difference between choosing 10 or 120 ?

Notice: same question for SUSY or non-SUSY models

NON-SUSY: 126 + 10

(2nd and 3rd generations only)

Bajc, A.M, Senjanovic, Vissani 2005

$$M_U = y_{10}\,\langle 1,2,2\rangle^u_{10} + y_{126}\,\langle 15,2,2\rangle^u_{126}$$
$$M_D = y_{10}\,\langle 1,2,2\rangle^d_{10} + y_{126}\,\langle 15,2,2\rangle^d_{126}$$
$$M_E = y_{10}\langle 1,2,2\rangle^d_{10} - 3\,y_{126}\,\langle 15,2,2\rangle^d_{126}$$
$$M_{\nu_D} = y_{10}\langle 1,2,2\rangle^u_{10} - 3\,y_{126}\,\langle 15,2,2\rangle^u_{126}$$

$$M_{\nu_L} = y_{126}\,\langle \overline{10},3,1\rangle^d_{126}$$
$$M_{\nu_R} = y_{126}\,\langle 10,1,3\rangle^d_{126}$$

see-saw, type I and II: $M_N = -M_{\nu_D} M^{-1}_{\nu_R} M_{\nu_D} + M_{\nu_L}$

approx. $\theta_q = V_{cb} = 0$

$$\frac{\langle 2,2,1\rangle^u_{10}}{\langle 2,2,1\rangle^d_{10}} = \frac{m_c(m_\tau - m_b) - m_t(m_\mu - m_s)}{m_s m_\tau - m_\mu m_b} \approx \frac{m_t}{m_b}$$

- real 10: $m_t = m_b$
- need a complex 10 - PQ symmetry → axion as Dark Matter

SUSY or not: 126 + 10

Bajc, Senjanovic, Vissani 2002

$$M_D = y_{10} \langle 1,2,2\rangle^d_{10} + y_{126} \langle 15,2,2\rangle^d_{126}$$
$$M_E = y_{10} \langle 1,2,2\rangle^d_{10} - 3\, y_{126} \langle 15,2,2\rangle^d_{126}$$

Type II see-saw: $M_N = M_{\nu_L} = y_{126} \langle 10,1,3\rangle^d_{126}$

$\theta_D = 0$ (small mixing in M_D)

$m_s = m_\mu = 0$

$$M_N \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix}$$

unless $m_b = m_\tau$, neutrino mixing vanishes

large $\theta_{atm} \leftrightarrow b - \tau$ unification

Full 3-gen. analysis:
- connection still true
- θ_{13} close to exp. limit

Matsuda, Koide, Fukuyama, Nishiura 2002

Goh, Mohapatra, Ng, 2003

NON-SUSY: 126 + 120

(2nd and 3rd generations only)

$$M_U = y_{120}(\langle 1,2,2\rangle^u_{120} + \langle 15,2,2\rangle^u_{120}) \quad +y_{126}\,\langle 15,2,2\rangle^u_{126}$$

$$M_D = y_{120}(\langle 1,2,2\rangle^d_{120} + \langle 15,2,2\rangle^d_{120}) \quad +y_{126}\,\langle 15,2,2\rangle^d_{126}$$

$$M_E = y_{120}(\langle 1,2,2\rangle^d_{120} - 3\langle 15,2,2\rangle^d_{120}) \quad -3\,y_{126}\,\langle 15,2,2\rangle^d_{126}$$

$$M_{\nu_D} = y_{120}(\langle 1,2,2\rangle^u_{120} - 3\langle 15,2,2\rangle^u_{120}) \quad -3\,y_{126}\,\langle 15,2,2\rangle^u_{126}$$

y_{120} antisymmetric

- real 120: $m_t = m_b$
- complex 120: interesting connections with neutrino masses and mixings

SUSY or not: 126 + 120

(2nd and 3rd generations only)

Bajc, A.M, Senjanovic, Vissani 2005

Defining some small ratios : $\epsilon_f = m_2^f / m_3^f$

Predictions:

- neutrino masses:

$$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{\cos 2\theta_A}{1 - \sin^2 2\theta_A/2} + \mathcal{O}(|\epsilon|)$$

  - large θ_A gives degenerate neutrinos

- quark masses relation at the GUT scale:

$$\frac{m_\tau}{m_b} = 3 + 3\sin 2\theta_A \ \mathrm{Re}[\epsilon_E - \epsilon_D] + \mathcal{O}(|\epsilon^2|)$$

  - but $m_\tau \approx 2m_b$. - include running, 3-gen. effects...

- quark mixing:

$$|V_{cb}| \simeq cos 2\theta_A \frac{m_s}{m_b} + O(|\epsilon^2|)$$

  - large neutrino mixing implies small quark mixing

126 + 10 or 126 + 120 ?

- In non-supersymmetric models, both posible in principle
  - 10 and 120 need to be complex
  - can have a PQ symmetry - axion as DM
- SUSY requires 126 + 10 for $m_b = m_\tau$
- Type II (even Type I) see-saw can give relations between neutrino and charged fermions masses and mixings
- Detailed models can be even more predictive: symmetry breaking and unification constraints
- 10 + 120 ? radiative see-saw - works for split-SUSY

Bajc, Senjanovic 2005

UNIFICATION: NON-SUSY

Deshpande, Keith, Pal 1993

$m_\nu \geq m_t^2/M_R$ → $M_R \geq 10^{13}$ GeV

		$\log(M_R/GeV)$
[redacted]		
II :	$SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[210]{} \{2_L 2_R 1_X 3_c P\} \xrightarrow[h]{} \{2_L 1_Y 3_c\}$	8.6 - 13.6
III :	$SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[45]{} \{2_L 2_R 1_X 3_c\} \xrightarrow[h]{} \{2_L 1_Y 3_c\}$	8.0 - 13.6
[redacted]		
VI :	$SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[45]{} \{2_L 1_R 4_C\} \xrightarrow[h]{} \{2_L 1_Y 3_c\}$	12.2 - 13.6
VII :	$SO(10) \xrightarrow[54]{} \{2_L 2_R 4_C P\} \xrightarrow[210]{} \{2_L 2_R 4_C\} \xrightarrow[h]{} \{2_L 1_Y 3_c\}$	11.3 - 13.6
[redacted]		

Unification: SUSY

- One-step: no intermediate scales
  - $m_\nu \propto M_W^2/M_{GUT}$ can be too small

- Potentials very constrained: no survival principle
  - calculate all the masses

- See-saw + SUSY = MSSM with R-parity
  - R-parity is in the center of SO(10)
  - R-parity $\equiv$ Matter parity = $(-1)^{3(B-L)}$
  - See-saw: break (B-L) with a (B-L)-even field in order to give ν_R a mass

Aulakh, A.M, Rasin, Senjanovic 1998

...get R-parity preserved
and the stable LSP is a DM candidate

What is the minimal renormalizable SUSY- GUT ?

- Based on SO(10)
- With a see-saw for neutrino mass: $\overline{126}$ (+ 126)
- Yukawa sector: 10 + $\overline{126}$ needed: the light Higgs must be a combination of doublets in 10 and $\overline{126}$
  - need a mixing $\langle\Phi\rangle\, H_{10}\, \overline{\Sigma}_{\overline{126}}$ can use 210

Babu, Mohapatra, 1993

- Symmetry breaking down to LR
(126, $\overline{126}$ break down to MSSM)

$$\Phi_{210}\,,\; H_{10}\,,\; \overline{\Sigma}_{\overline{126}}\,,\; \Sigma_{126}$$

  - 210 can do that too

MINIMAL SO(10)

Clark, Kuo, Nakagawa,1982

Aulakh, Bajc, A.M, Senjanovic, Vissani,2003

$$\Psi_{16}, H_{10}, \Sigma_{126}, \overline{\Sigma}_{\overline{126}}, \Phi_{210}$$

$$\begin{aligned} W_H &= m_\Phi \Phi^2 + m_\Sigma \Sigma\overline{\Sigma} + \lambda\Phi^3 + \eta\Phi\Sigma\overline{\Sigma} + m_H H^2 + \Phi H(\alpha\Sigma + \bar{\alpha}\overline{\Sigma}) \\ &+ y_{10}\Psi C\Gamma\Psi H + y_{126}\Psi C\Gamma^5\Psi\overline{\Sigma} \end{aligned}$$

- 26 real parameters: same as MSSM
- light Higgs made up of 126, 10 and 210 doublets
  - rich enough Yukawa structure
- Type I and II see-saw
  - possibility of connecting large θ_A with $b-\tau$ unification
- symmetry can be broken down to MSSM (+R-parity)
  - stable LSP

SYMMETRY BREAKING

$$H \equiv \mathbf{10} = (6,1,1) + (1,2,2)$$

$$\Phi \equiv \mathbf{210} = (15,1,1) + (1,1,1) + (15,1,3)$$
$$+ (15,3,1) + (6,2,2) + (10,2,2) + (\overline{10},2,2)$$

$$\Sigma \equiv \mathbf{126} = (\overline{10},1,3) + (10,3,1) + (6,1,1) + (15,2,2)$$

$$\overline{\Sigma} \equiv \overline{\mathbf{126}} = (10,1,3) + (\overline{10},3,1) + (6,1,1) + (15,2,2)$$

doublets: vev $\sim M_W$

SM singlets: vev $\sim M_{GUT}$

type II see-saw: vev $\sim M_W^2/M_{GUT}$

- Find the symmetry breaking conditions
- Calculate masses for all states
- Find the composition of the light Higgs doublets

Bajc, A.M, Senjanovic, Vissani 2004

Aulakh, Girdaar, 2004

Fukuyama, et. al. 2004

An overconstrained model

After fine-tune of the SM Higgs mass:
8 parameters left in the heavy Higgs sector

$$m, \alpha, \overline{\alpha}, |\lambda|, |\eta|, \phi = \arg\lambda = -\arg\eta \qquad x = \Re(x) + i\Im(x)$$

ratio of masses

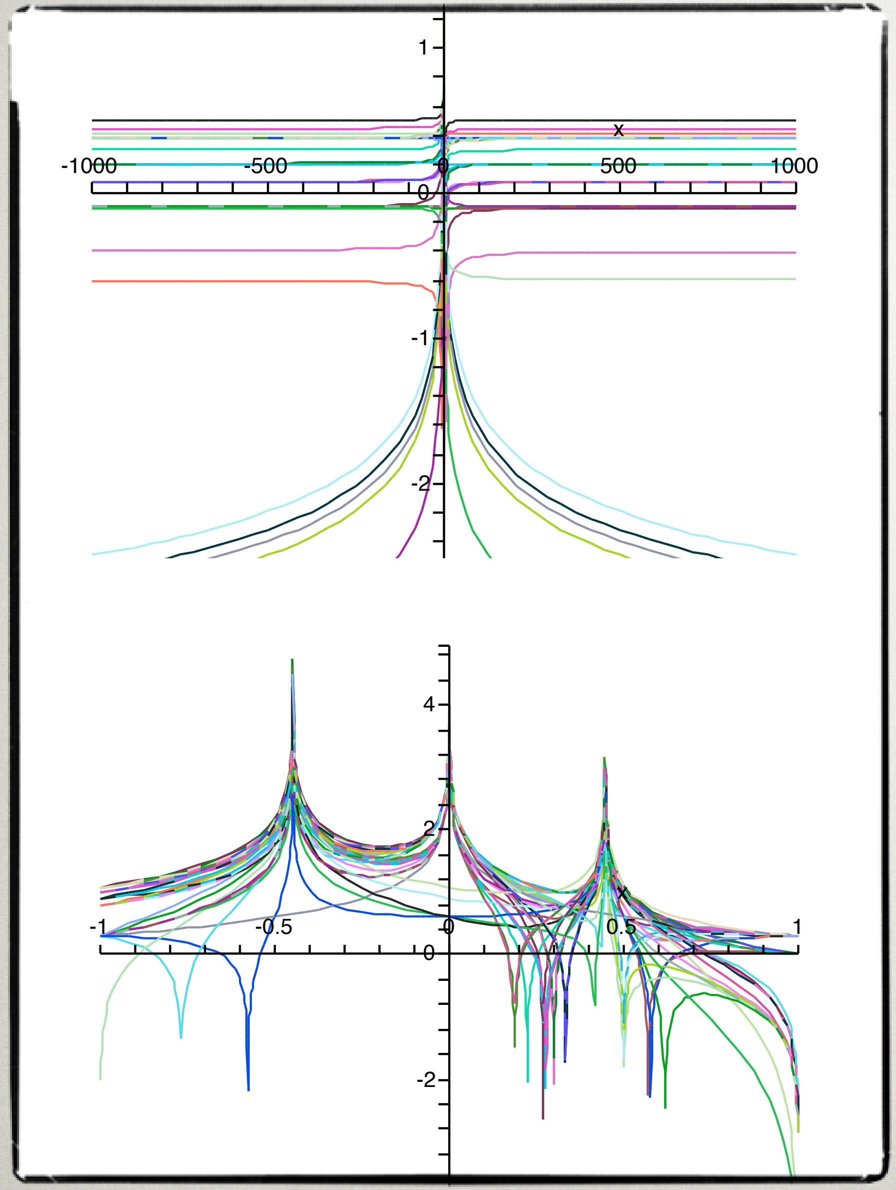
Vevs and masses of all states have form:

$$\sim \frac{m}{\lambda} f(x)$$

$$\frac{m}{\sqrt{\lambda\eta}} f(x)$$

- variation with parameters quite smooth, with x non -trivial

Masses of all states



$$Log[M_i/10^{16}]$$

$x \rightarrow \infty$

x real < 1

Light states spoil unification:
keep x < 1

Fermion mass fitting

- The light Higgs is a combination no longer arbitrary

$$H_{u,d} = r^{\mathbf{10}}_{u,d} H^{\mathbf{10}}_{u,d} + r^{\overline{\mathbf{126}}}_{u,d} H^{\overline{\mathbf{126}}}_{u,d} + r^{\mathbf{126}}_{u,d} H^{\mathbf{126}}_{u,d} + r^{\mathbf{210}}_{u,d} H^{\mathbf{210}}_{u,d}$$

  - $r^{\mathbf{I}}_{u,d}$ known functions of the parameters

- Assume type II see-saw

$$m_\nu = y_{126}\, v_\Delta \qquad v_\Delta = \frac{(\alpha r^{\mathbf{10}}_u + \sqrt{6}\eta r^{\overline{\mathbf{126}}}_u) r^{\mathbf{210}}_u}{m_\Delta}$$

  - neutrino mass depends on the same parameters

Trouble for Type II see-saw

Some relations among fermion masses depend only on x

$$M_u = \frac{N_u}{N_d} \tan\beta \times [M_d + \xi(x)(M_d - M_e)]$$

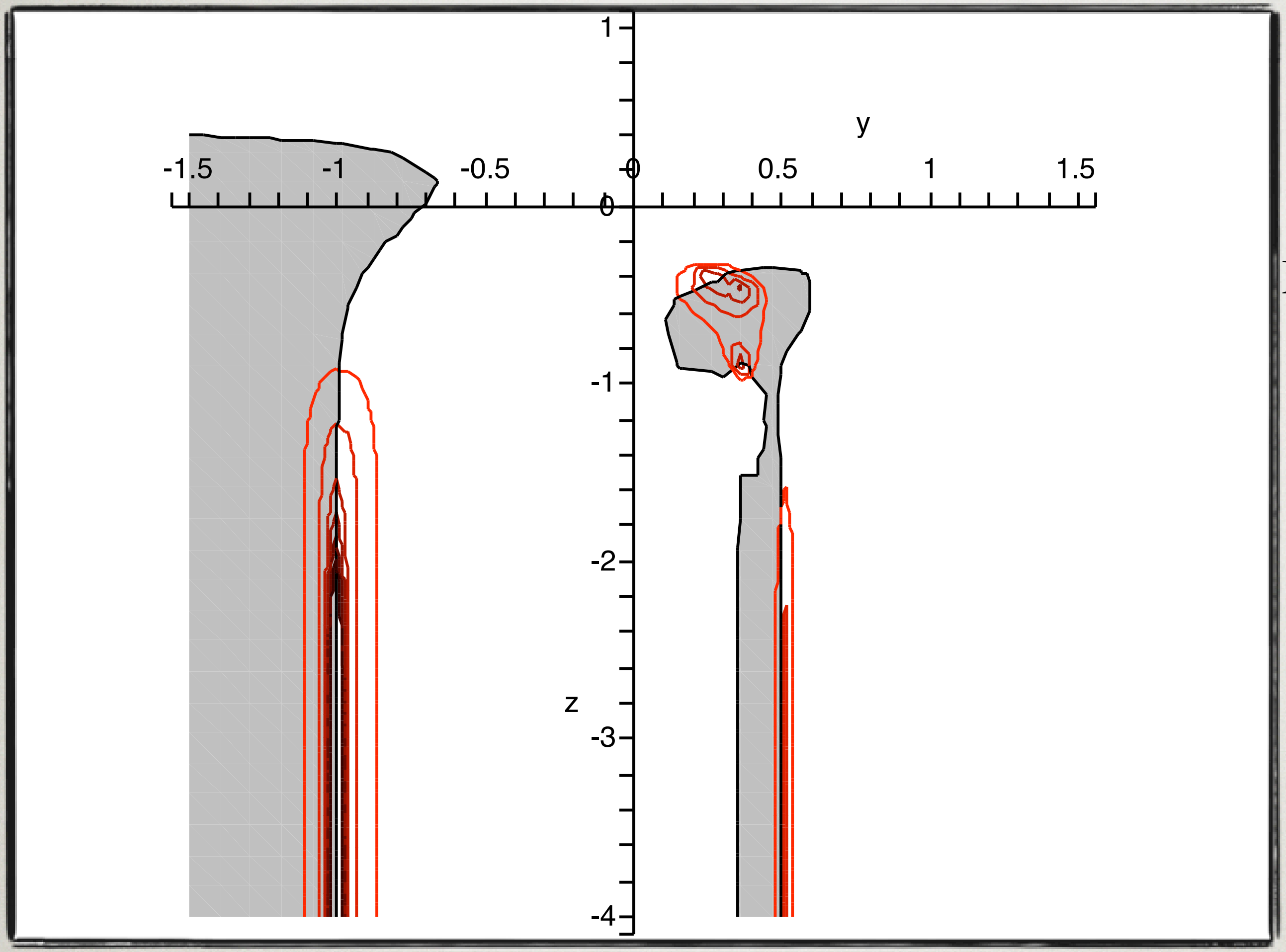
Define the ratio $R = |1 + 1/\xi(x)|$

→ then $R > 1$ from trace identities

Write type-II mass as

$$m_{II} = \frac{v^2}{M_x} \times \frac{\sin^2\beta}{\cos\beta} \times \alpha\sqrt{\frac{|\lambda|}{|\eta|}} \times \frac{M_d - M_e}{v} \times \frac{N_u^2}{N_d}\xi_{II}(x)$$

→ then $\xi_{II}(x)$ must give $10^2 - 10^3$



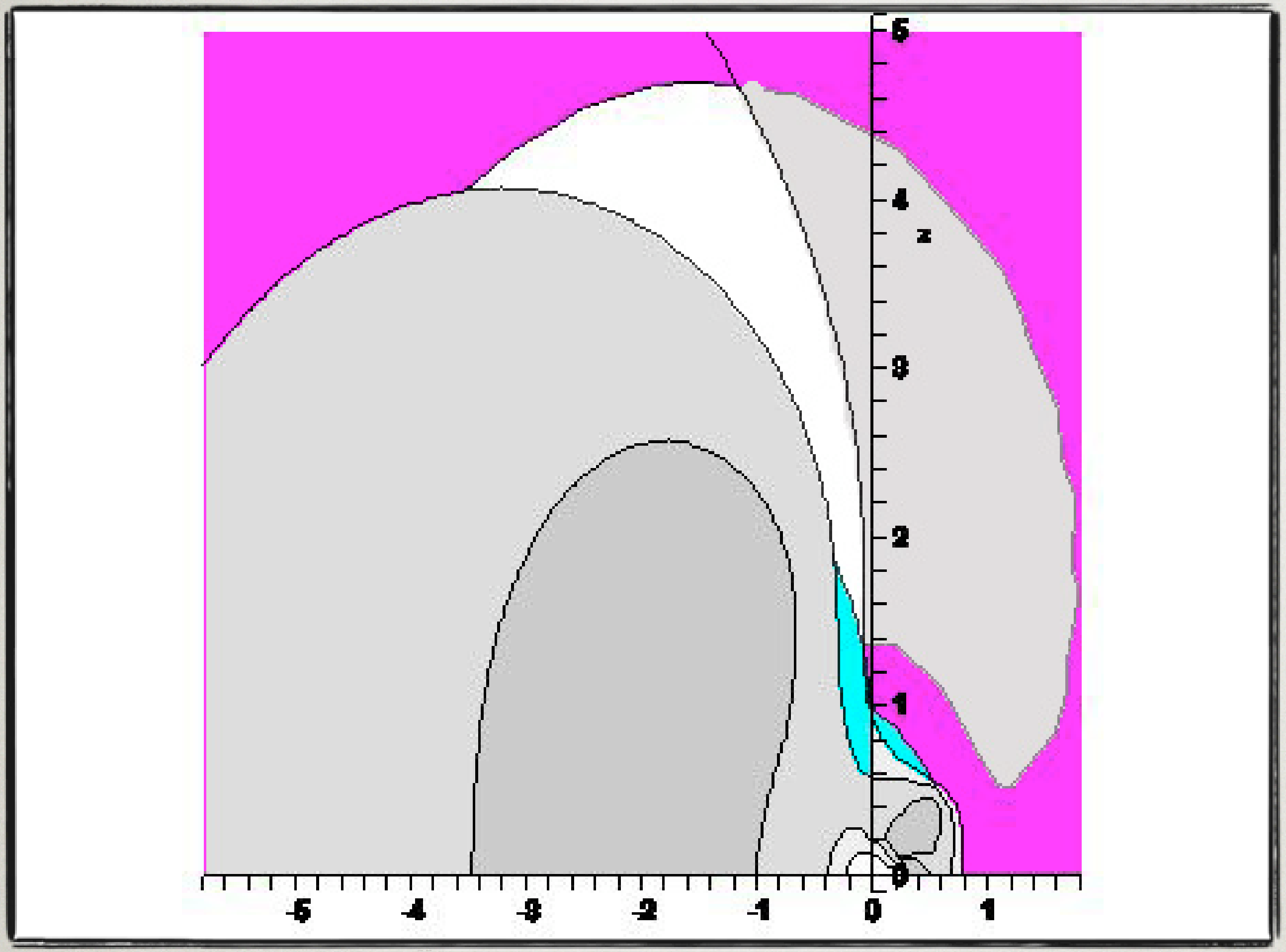
Maxima of $\frac{N_u^2}{N_d}\xi_{II}(x)$ (red contours)

regions with $R > 1$ (white)

too large yukawa coup.

TOO SMALL NEUTRINO MASS

Bajc, A.M, Senjanovic, Vissani 2005



THRESHOLD EFFECTS

- $\Delta \sin^2 \theta_W \quad (M_W)$
- $\Delta \alpha_U \quad (M_{GUT})$
- $R > 1$
- $R > 2$

General analysis (type I and II)

Bertolini, Frigerio, Malinsky, 2005-2006

Aulakh,Garg, Girdaar, 2005-2006

Mohapatra, Goh, Ng, Dutta, Mimura...

Babu, Macesanu

Wang, Yang

- Do the complete fit with all fermion masses and all parameters
- Parameter space for type I and type II getting smaller
- Include unification constrains, threshold effects - even worse

too small neutrino mass:
model seems to be ruled out !

What to do?

Add more Yukawa couplings: 120

Aulakh, 2005-2006

$$D_{120} \;=\; (\overline{10},1,1)+(10,1,1)+(6,3,1)+(6,1,3)+(1,2,2)+(15,2,2)$$

(another 10 or 126 cannot help)

- No SM singlets: symmetry breaking is the same
- Antisymmetric: only 3 complex Yukawa couplings more
- Two doublets mix through:

$$c_1 D_{120} H_{10} \Phi_{210} + c_2 D_{120} \Sigma_{126} \Phi_{210} + c_3 D_{120} \overline{\Sigma}_{126} \Phi_{210}$$

- More parameters in the superpotential

$$m_D\,,\; c_1\,,\; c_2\,,\; c_3\,,\; y_{120} \qquad 26+13=39$$

Or: change the Higgs sector

Alternate model: 54 + 45 instead of 210

S : 54
A : 45
Σ : 126
H : 10

$$W = m_H H^2 + m_S S^2 + m_A A^2 + m_\Sigma \Sigma\overline{\Sigma} + \eta A \Sigma \overline{\Sigma}$$
$$+ \lambda_H H^2 S + \lambda_S S^3 + \lambda_A A^2 S + \lambda_\Sigma \Sigma^2 S + \lambda_{\overline{\Sigma}} \overline{\Sigma}^2 S$$

- 29 real parameters

  (compare to 26 in minimal model with 210)

- see-saw of type I and II
- 10 + 126 but...
  - they do not mix - light Higgs is only 10

gives wrong fermion masses

54 + 45 with added 120: all doublets mix

$$DAH + DA\Sigma + DA\overline{\Sigma}$$

D : 120
H : 10
A : 45
Σ : 126

Compare with minimal model

- once you have to enlarge the Yukawa sector, almost same number of parameters: 41 vs. 39
- smaller representations

➡ find symmetry breaking and mass spectrum

Ramírez, A.M., in prep.

Symmetry breaking

$$\begin{aligned}
\langle(1,1,1)_{\mathbf{54}}\rangle \equiv s &= \frac{m_A}{\lambda_A}\, x \\
\langle(1,1,15)_{\mathbf{45}}\rangle \equiv a &= \frac{m_\Sigma}{\eta_A}\frac{6x-1}{1-2x} \\
\langle(1,3,1)_{\mathbf{45}}\rangle \equiv b &= \frac{m_\Sigma}{\eta_A}\frac{4x+1}{1-2x} \\
\langle(1,3,10)_{\mathbf{126}}\rangle = \langle(1,3,\overline{10})_{\overline{\mathbf{126}}}\rangle \equiv \sigma &= \sqrt{\frac{m_\Sigma m_A}{\eta_A^2}\frac{(6x-1)(4x+1)}{1-2x}}
\end{aligned}$$

with

$$\left(\frac{2}{5}\frac{m_\Sigma}{m_A}\frac{\lambda_A}{\eta_A}\right)^2 \frac{x-1}{(1-2x)^2} - \frac{\lambda_S}{\lambda_A}x = \frac{m_S}{m_A}$$

- one-step breaking at $10^{16} GeV$
- can arrange for a Type II see-saw dominance

Example: Arranging a light triplet

RGE in the MSSM

$$\ln\left(\frac{M_{GUT}}{M_W}\right) = \left(\frac{1}{\alpha_j} - \frac{1}{\alpha_i}\right)\frac{2\pi}{b_i - b_j} \qquad i = 1, 2, 3$$

Suppose the Δ triplet has a mass $< M_{GUT}$

$$v_\Delta \propto \frac{1}{m_\Delta} \qquad m_\nu = y_{126}\, v_\Delta$$

Type II see-saw

other light fields could cancel its contribution to the running

$SU(3) \times SU(2) \times U(1)$	δb_1	δb_2	δb_3
$(1, 3; \pm 1)$ Δ	9/5	2	0
$(6, 1; \pm 1/3)$	2/5	0	5/2
$(1, 2; \pm 1/2)$	3/10	1/2	0
Total	5/2	5/2	5/2

All these fields are available

Δ

$$(S_{133^-}, \Sigma_{131}) \quad (S_{133^+}, \overline{\Sigma}_{\overline{1}31}) \qquad \begin{pmatrix} m_S + 6\lambda_S s & -\sqrt{2}\lambda_\Sigma \sigma \\ \sqrt{2}\lambda_{\overline{\Sigma}}\overline{\sigma} & m_\Sigma - 3\eta a \end{pmatrix}$$

$$(\overline{\Sigma}_{613^0}, C_{611}) \quad (\Sigma_{\overline{6}13^0}, C_{\overline{6}11}) \qquad \begin{pmatrix} m_\Sigma - \eta\, a & -\sqrt{2}\,\beta\, b \\ \sqrt{2}\,\overline{\beta}\, b & m_C - 12\lambda_C\, s \end{pmatrix}$$

$$(H_{122^-}, \Sigma_{122^-}, \overline{\Sigma}_{122^-}, C_{122^-}, C_{1'22^-})$$
$$(H_{122^+}, \overline{\Sigma}_{122^+}, \Sigma_{122^+}, C_{122^+}, C_{1'22^+})$$

$$\begin{pmatrix} m_H + 3\lambda_H\, s & 0 & 0 & \alpha\, b & -\sqrt{3}\,\alpha\, a \\ 0 & m_\Sigma - \eta\, b & -5\lambda_\Sigma s & \sqrt{\frac{3}{2}}\,\overline{\beta}\, a & -\frac{1}{\sqrt{2}}\,\overline{\beta}\,(2a-b) \\ 0 & -5\lambda_\Sigma s & m_\Sigma + \eta\, b & -\sqrt{\frac{3}{2}}\,\beta\, a & -\frac{1}{\sqrt{2}}\beta\,(2a+b) \\ \alpha\, b & \sqrt{\frac{3}{2}}\,\beta\, a & -\sqrt{\frac{3}{2}}\,\overline{\beta}\, a & m_C + 18\lambda_C s & 0 \\ \sqrt{3}\,\alpha\, a & \frac{1}{\sqrt{2}}\,\beta\,(2a-b) & \frac{1}{\sqrt{2}}\,\overline{\beta}\,(2a+b) & 0 & m_C - 2\lambda_C s \end{pmatrix}$$

- enough free parameters to tune their masses at an intermediate scale
- triplet can be as light as necessary for neutrino mass without affecting unification constraints

Summary

- SO(10): ideal framework for small neutrino mass
- Models can provide connections between fermion masses and mixings, for example
  - $b - \tau$ unification ⟺ large θ_{atm} (10+126)
  - large neutrino ⟺ small quark mixings (120+126)
  - large θ_{atm} ⟺ degenerate neutrinos (120+126)
- Non supersymmetric models are alive and well
- Minimal SUSY GUT is in trouble
  - ✳ lack of intermediate scales
- Next-to-minimal SUSY GUT may not be predictive ...
  - ✳ but work is in progress