Constraining the 2HDM parameter space

BW2007, September 2007

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work with A. El Kaffas, O. M. Øgreid,
arXiv:0706.2997

Bottom line:

Parameter space very constrained

Define model: 2HDM (II)

Potential:

$$
\begin{aligned}
V = {} & \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \\
& - \frac{1}{2}\left\{m_{11}^2(\Phi_1^\dagger\Phi_1) + \left[m_{12}^2(\Phi_1^\dagger\Phi_2) + \text{h.c.}\right] + m_{22}^2(\Phi_2^\dagger\Phi_2)\right\}
\end{aligned}
$$

Allow CP violation:

$\lambda_5,\ \lambda_6,\ \lambda_7,\ m_{12}^2$ may be complex

Neutral sector: 3 × 3 mixing matrix $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$

$$-\frac{\pi}{2} < \alpha_i \le \frac{\pi}{2}, \quad i = 1, 2, 3$$

Today: $\lambda_6 = \lambda_7 = 0$

Rotation matrix

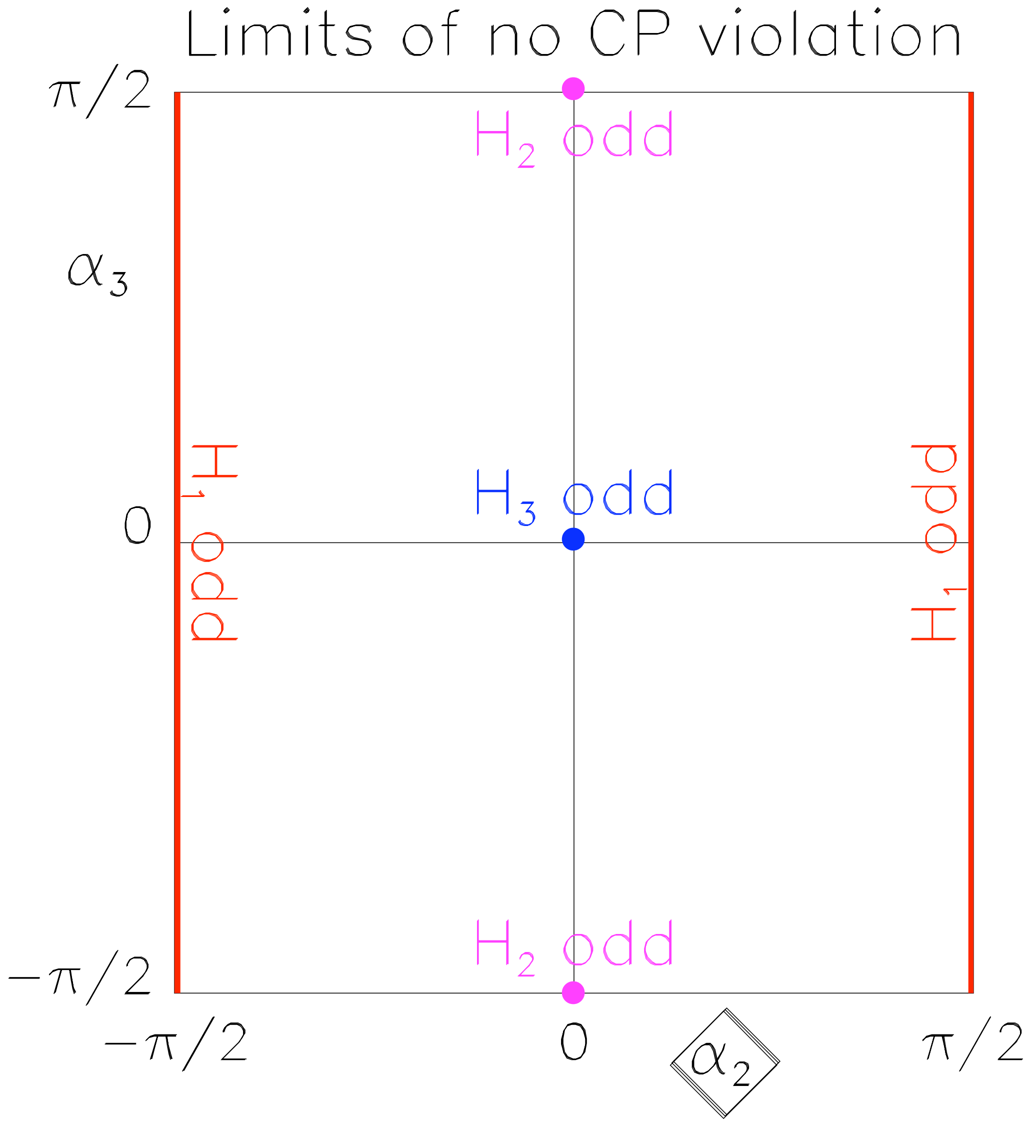
$$R\mathcal{M}^2R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

Mass squared

3 angles

$$R = \begin{pmatrix} c_1\,c_2 & s_1\,c_2 & s_2 \\ -(c_1\,s_2\,s_3+s_1\,c_3) & c_1\,c_3-s_1\,s_2\,s_3 & c_2\,s_3 \\ -c_1\,s_2\,c_3+s_1\,s_3 & -(c_1\,s_3+s_1\,s_2\,c_3) & c_2\,c_3 \end{pmatrix}$$

$$c_i = \cos\alpha_i, \;\; s_i = \sin\alpha_i$$


Limits of no CP violation
π/2
α3
0
−π/2
H2 odd
H1 odd
H3 odd
H1 odd
H2 odd
−π/2
0
α2
π/2

Parameters:

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \mu^2, \tan\beta$$

complex

or:

$$M_1 \leq M_2 \leq M_3, M_{H^\pm}, \tan\beta, \mu^2, \alpha_1, \alpha_2, \alpha_3$$

Input parameters:

$$M_1 \leq M_2, M_{H^\pm}, \tan\beta, \mu^2, (\alpha_1, \alpha_2, \alpha_3)$$

Calculate: $M_3, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$

Conditions: $M_2 \leq M_3$

$$V(\Phi_1, \Phi_2) > 0 \quad |\Phi_i| \to \infty$$

Special property

Neutral sector mass matrix (squared)
given by second derivatives of potential

For $\mathrm{Im}\,\lambda_5 \neq 0$

have $\mathcal{M}^2_{13} = \tan\beta\,\mathcal{M}^2_{23}$

$$\Rightarrow \boxed{\textstyle\sum_k M_k^2 R_{k3}(R_{k1} - R_{k2}\tan\beta) = 0}$$

Determine M_3 from $M_1 \leq M_2, \tan\beta, (\alpha_1, \alpha_2, \alpha_3)$

Yukawa couplings (Model II)

$$H_j b\bar{b} : \qquad \frac{1}{\cos\beta}\left[R_{j1} - i\gamma_5 \sin\beta R_{j3}\right]$$

$$H_j t\bar{t} : \qquad \frac{1}{\sin\beta}\left[R_{j2} - i\gamma_5 \cos\beta R_{j3}\right]$$

$$H^+ b\bar{t} : \qquad \frac{ig}{2\sqrt{2}m_W}\left[m_b(1+\gamma_5)\tan\beta + m_t(1-\gamma_5)\cot\beta\right]$$

$$H^- t\bar{b} : \qquad \frac{ig}{2\sqrt{2}m_W}\left[m_b(1-\gamma_5)\tan\beta + m_t(1+\gamma_5)\cot\beta\right]$$

Important at low tanβ

Constraints (three killers):

- Positivity
- Perturbative unitarity
- Experimental constraints

Experimental constraints:

Independent of neutral sector

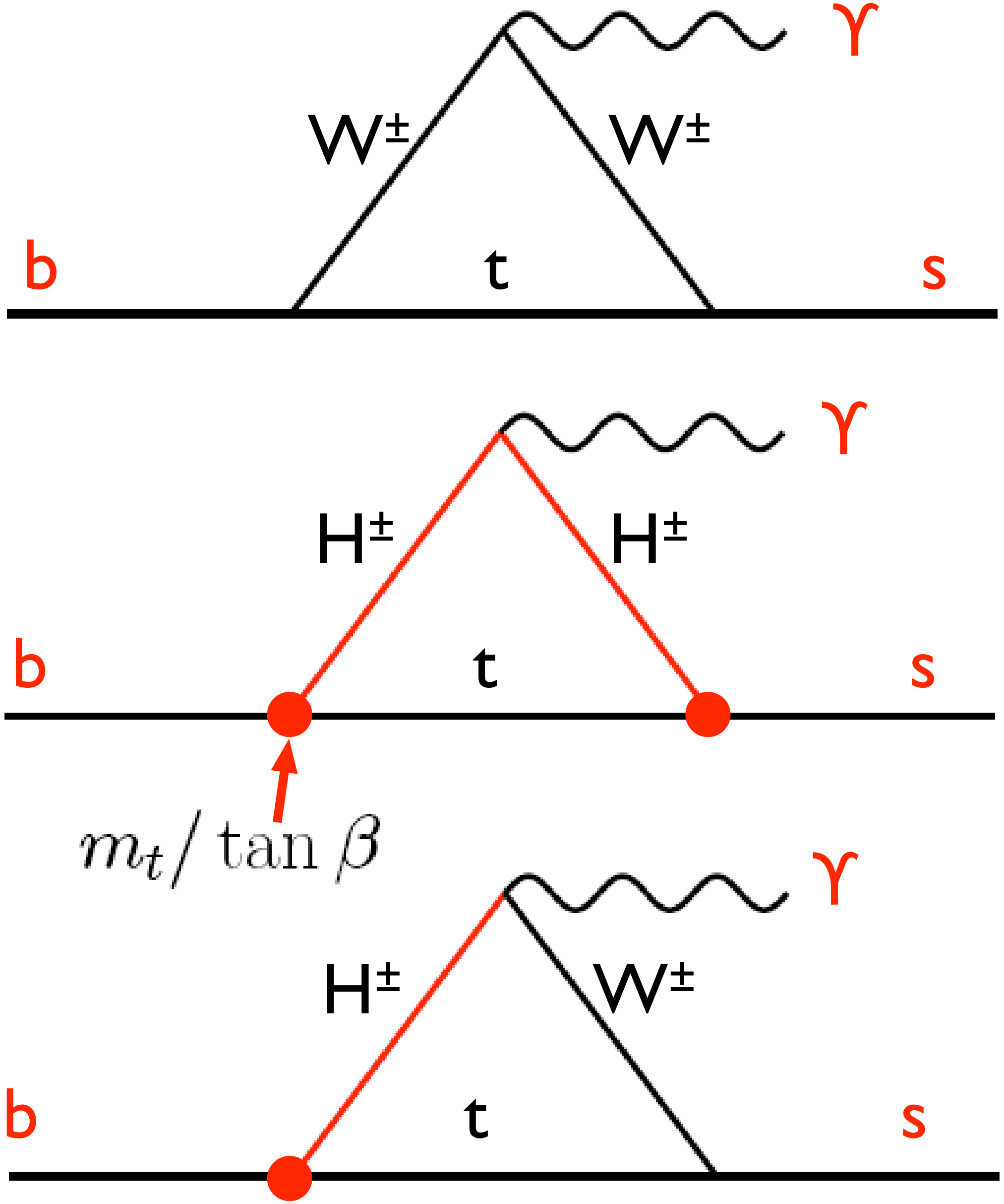
• $B \to X_s\gamma$	excludes low $M_{H^\pm}$
• $B \to \bar{B}$ oscillations	excludes low $\tan\beta$
• $B \to \tau\nu$	excludes high $\tan\beta$, low $M_{H^\pm}$

Depend on neutral sector

• $\Gamma_{Z \to b\bar{b}}$	excludes low $\tan\beta$
• LEP2 non-discovery	light H decouples
• $\Delta\rho$	spectrum compact
• $(g\text{-}2)_\mu$	rel. only at very large $\tan\beta$

$B \to X_s\gamma$

Sensitive to BSM physics



B → $X_s\gamma$ Misiak et al, 2006 (NNLO):

$$\mathcal{B}(\bar{B} \to X_s\gamma) = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{\mathrm{e.m.}}}{\pi C} \{P(E_0) + N(E_0)\} \times \mathcal{B}(\bar{B} \to X_c e \bar{\nu}_e)_{\mathrm{exp}}$$

$$\left|C_7^{(0)\mathrm{eff}}(\mu_b)\right|^2$$ at LO

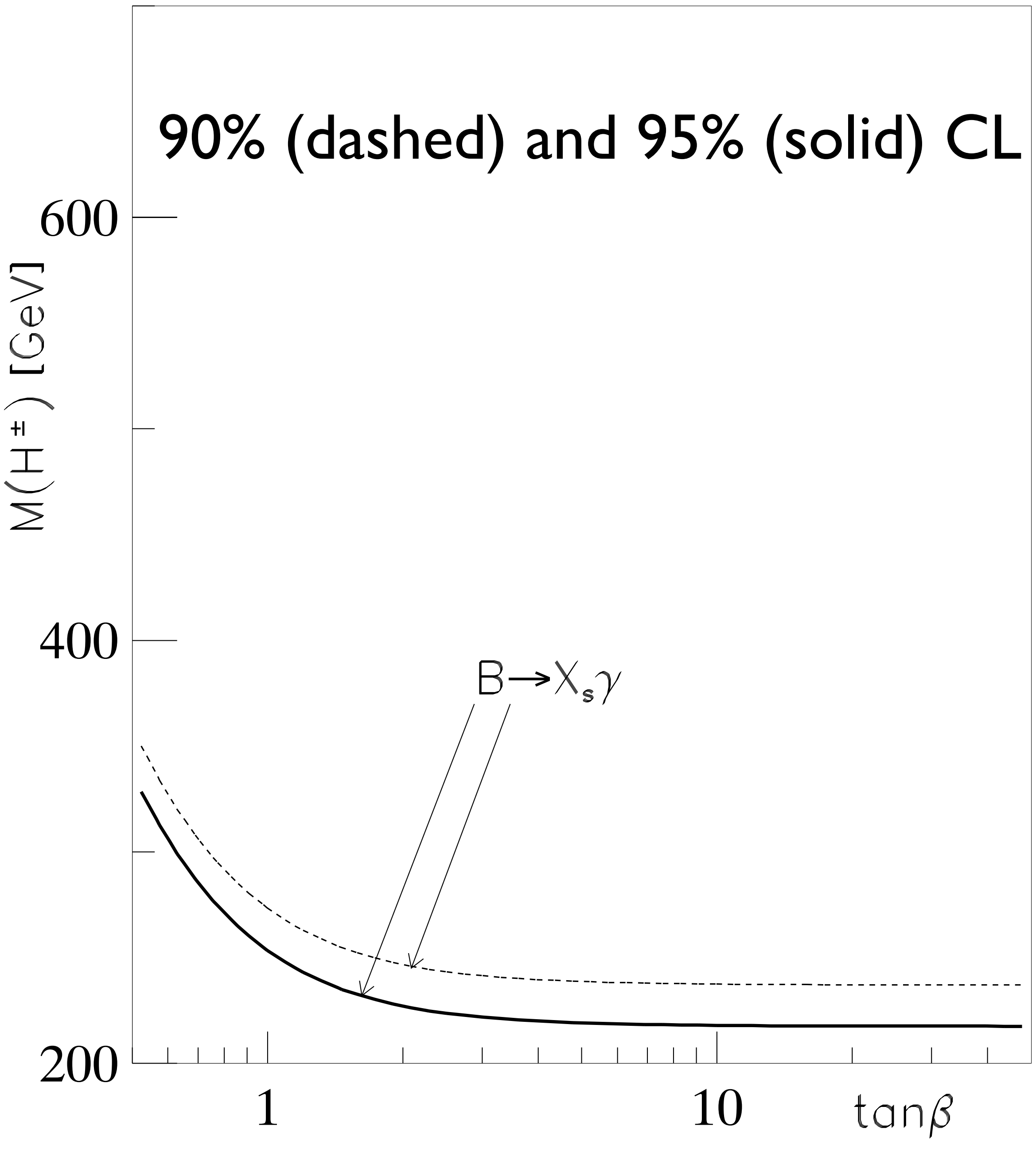
Misiak et al: $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$

HFAG (exp): $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.55 \pm 0.24 \pm \cdots) \times 10^{-4}$

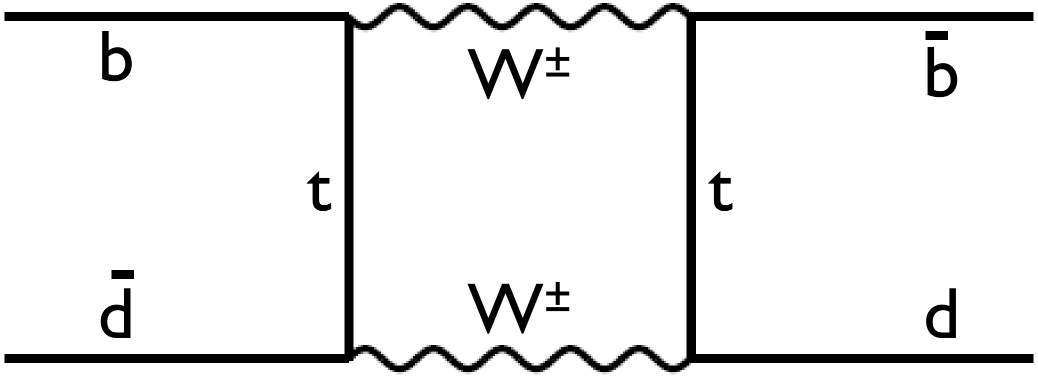
Measure of discrepancy:

$$\boxed{\chi^2_{b\to s\gamma} = \frac{[\mathcal{B}(\bar{B} \to X_s\gamma)_{\mathrm{2HDM}} - \mathcal{B}(\bar{B} \to X_s\gamma)_{\mathrm{ref}}]^2}{\{\sigma[\mathcal{B}(\bar{B} \to X_s\gamma)]\}^2}}$$

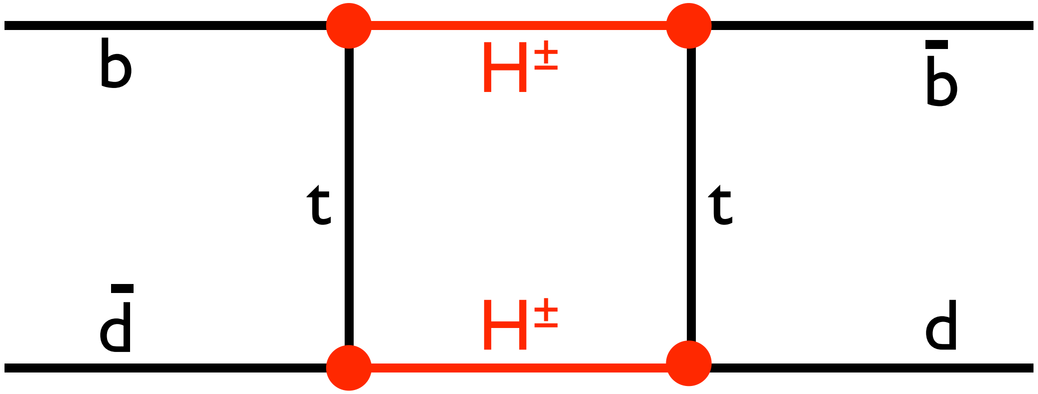
$$\sigma[\mathcal{B}(\bar{B} \to X_s\gamma)] = 0.35 \times 10^{-4}$$

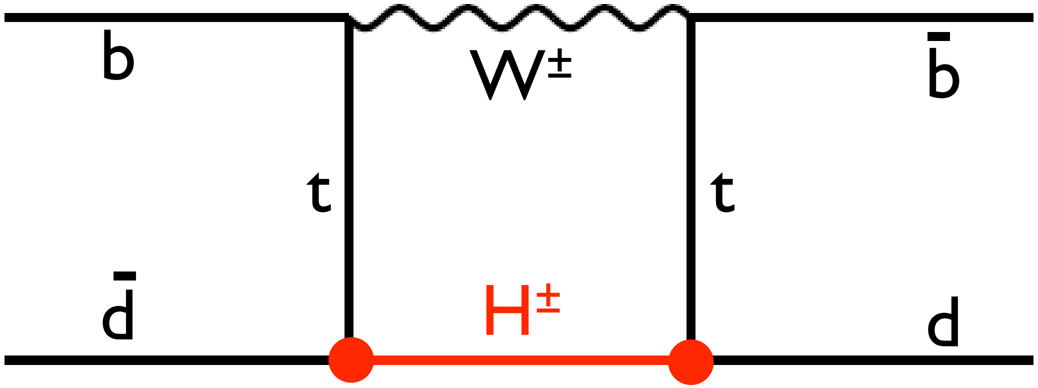

90% (dashed) and 95% (solid) CL
600
400
200
M(H±) [GeV]
B→X_sγ
1
10
tanβ

$B \to \bar{B}$



Sensitive to
BSM physics





B → B̄ Urban, Krauss, Jentschura, Soff 1998 (NLO):

$$\boxed{x_d \equiv \frac{\Delta m_{B_d}}{\Gamma_B} = \frac{G_F^2}{6\pi^2}|V_{td}^*|^2|V_{tb}|^2 f_B^2\, \underset{\uparrow}{B_B}\, m_B\, \underset{\uparrow}{\eta}\, \tau_B\, M_W^2\, S_{2HDM}}$$

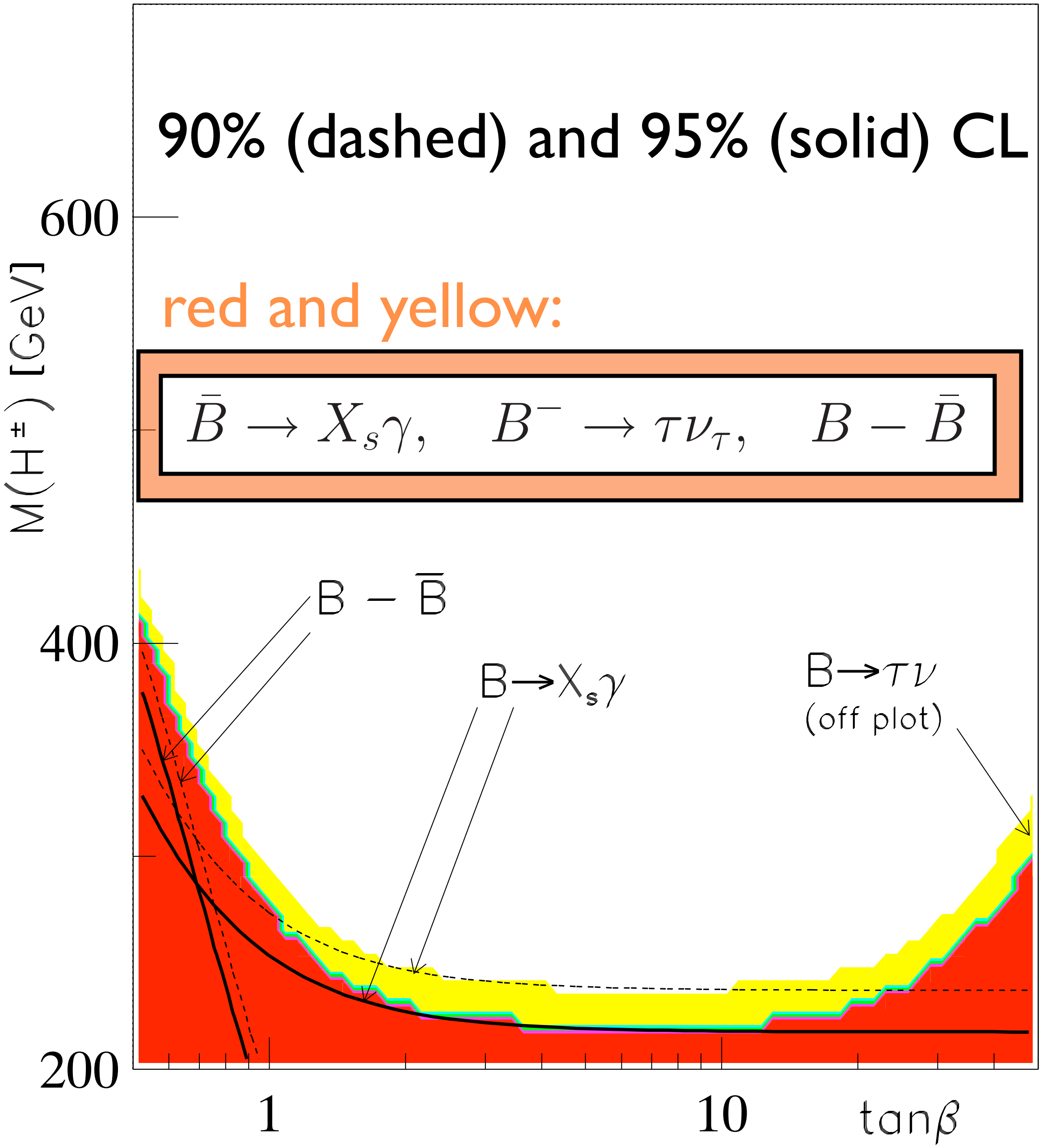
QCD corrections

Inami-Lim functions:

$$S_{2HDM} = S_{WW} + 2\underset{\substack{\uparrow \\ 1/\tan^2\beta}}{S_{WH}} + \underset{\substack{\uparrow \\ 1/\tan^4\beta}}{S_{HH}}$$

from Higgs Yukawa couplings

$\eta \times S_{2HDM}$ has weaker dependence on $\tan\beta$ and $M_{H^\pm}$ than S_{2HDM} 2.7 vs. 4.5



90% (dashed) and 95% (solid) CL

red and yellow:

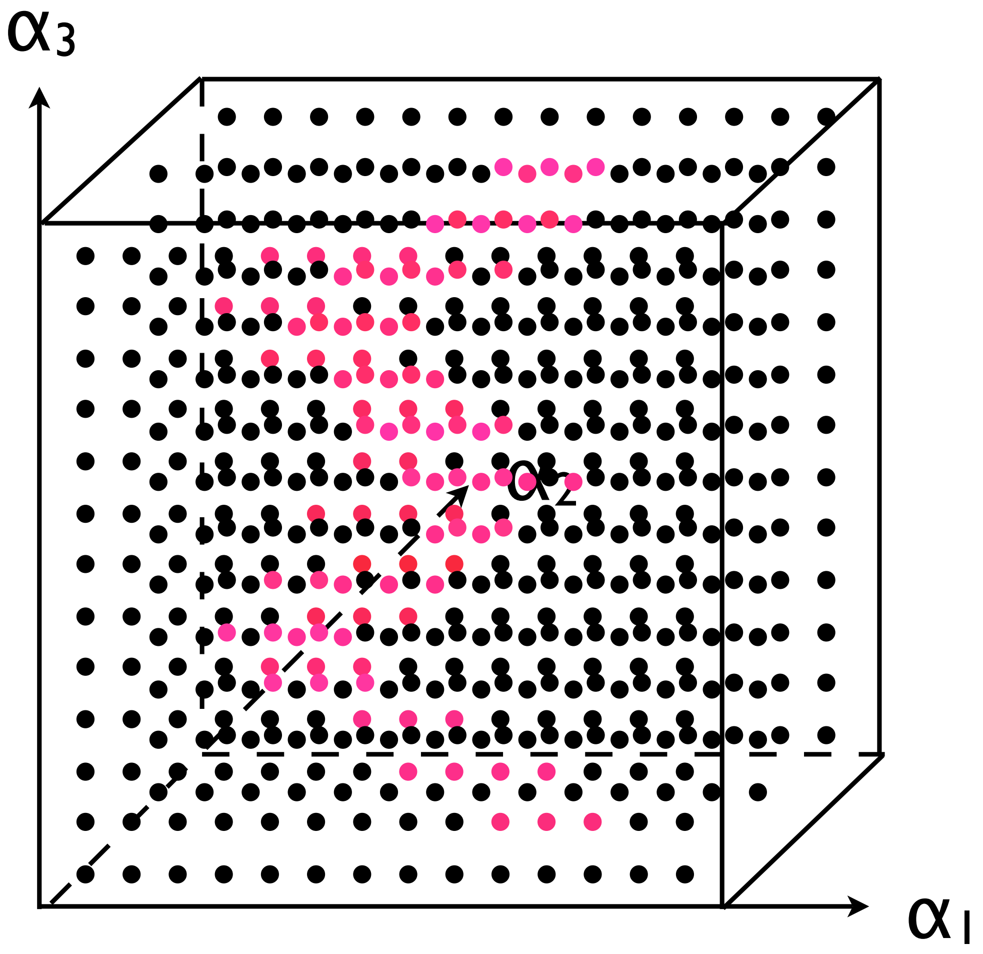
$$\boxed{\bar{B} \to X_s\gamma, \quad B^- \to \tau\nu_\tau, \quad B - \bar{B}}$$

combined

Consistency of Neutral Sector

Positivity

- Choose $M_1 \le M_2,\ \mu^2$
- Loop over $\tan\beta,\ M_{H^\pm}$
- For each $\tan\beta, M_{H^\pm}$ scan over $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$

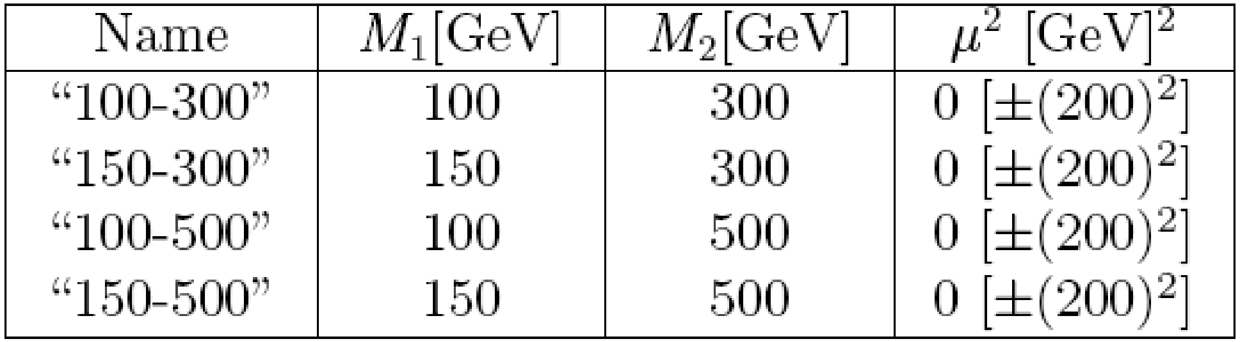

α3
α2
α1

Consistency of Neutral Sector

Positivity

- Choose $M_1 \leq M_2, \ \mu^2$
- Loop over $\tan\beta, \ M_{H^\pm}$
- For each $\tan\beta, M_{H^\pm}$ scan over $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$
- Count fraction of points where positivity is satisfied.
- Result: about 20%, denote these points $\boldsymbol{\alpha}_+$

Reference (mass) points



Name	M_1[GeV]	M_2[GeV]	μ^2 [GeV]^2
"100-300"	100	300	0 $[\pm(200)^2]$
"150-300"	150	300	0 $[\pm(200)^2]$
"100-500"	100	500	0 $[\pm(200)^2]$
"150-500"	150	500	0 $[\pm(200)^2]$

Table 1: Reference masses.

Recall: $M_1 \leq M_2 \leq \underset{\uparrow}{M_3}$

calculated from $(\alpha_1, \alpha_2, \alpha_3)$

Unitarity

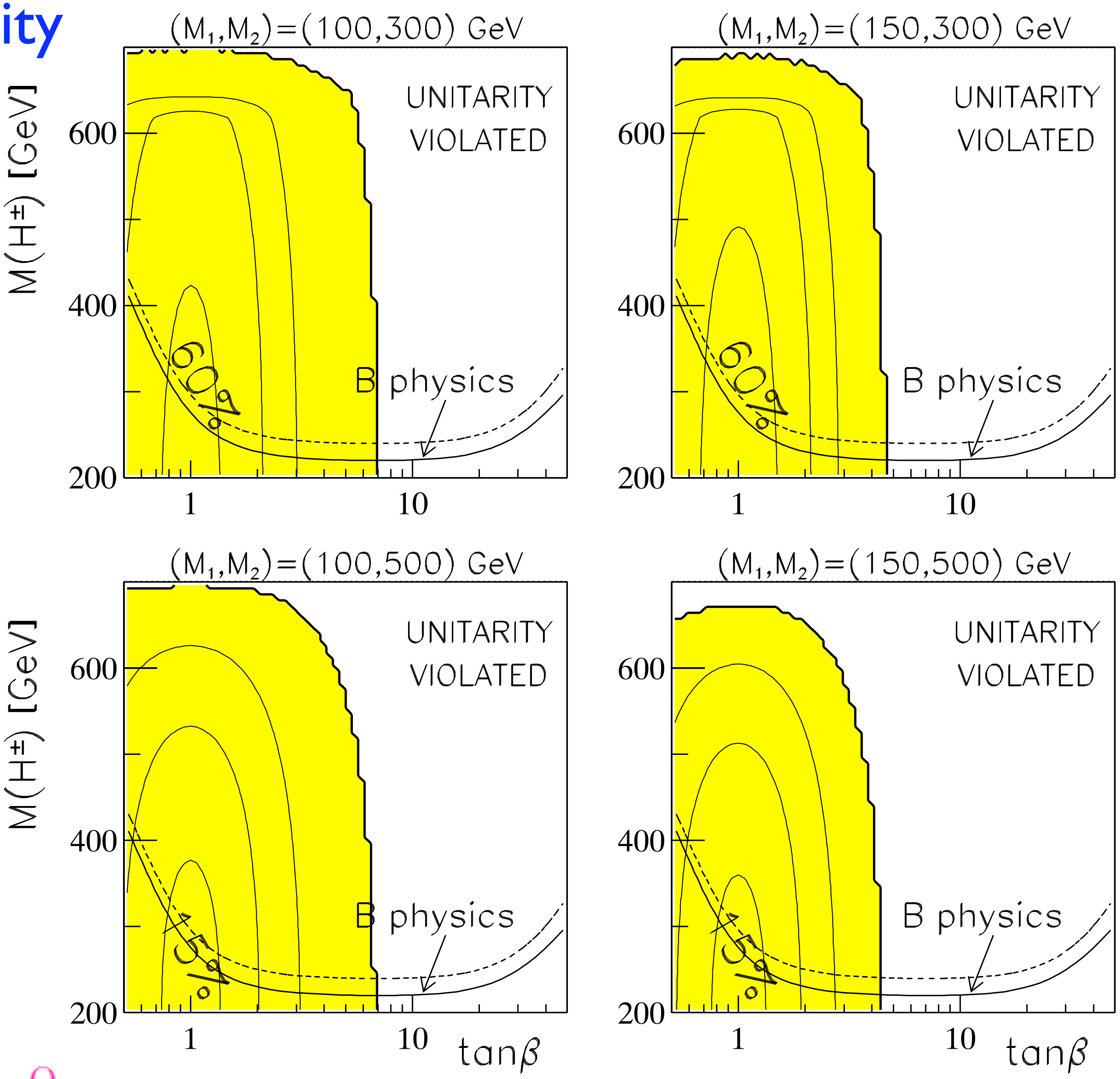
Higgs-Higgs scattering

Kanemura, Kubota, Takasugi (1993);
Akeroyd, Arhrib, Naimi (2000);
Ginzburg, Ivanov (2003, 2005)

Now focus on region not excluded by charged-Higgs constraints

- Choose $M_1 \leq M_2, \quad \mu^2$
- Loop over $\tan\beta, \ M_{H^\pm}$
- Scan over $\boldsymbol{\alpha}_+$
- Count fraction of points where unitarity is satisfied.
- Result: up to 60% $\hat{\boldsymbol{\alpha}} \in \boldsymbol{\alpha}_+ \in \boldsymbol{\alpha}$
- Peaked at low $\tan\beta$
- Cut off at high $\tan\beta$ and high $M_{H^\pm}$

Unitarity



Isospin 0, hypercharge 0 channel most restrictive at high tanβ

$\mu^2 = 0$

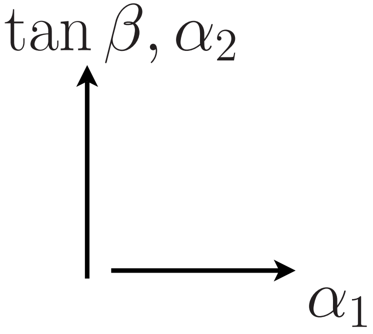
Note cut-off at ‘large’ tan β

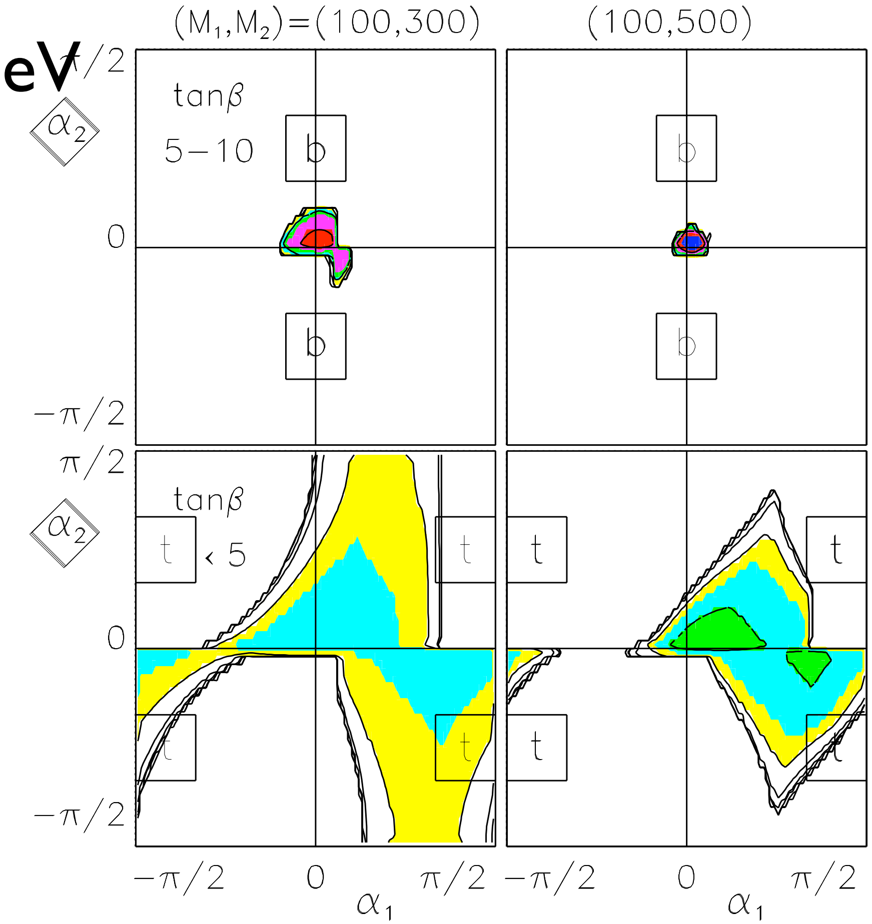
Regions in (α_1, α_2) populated by allowed (by unitarity) solutions

M1, M2=100, 300 & 500 GeV

μ=0

2 slices of tanβ



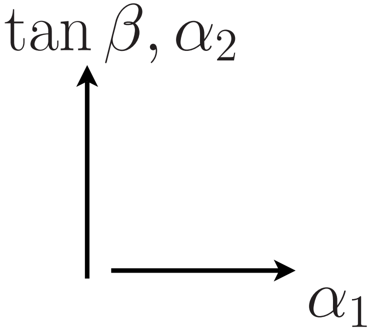


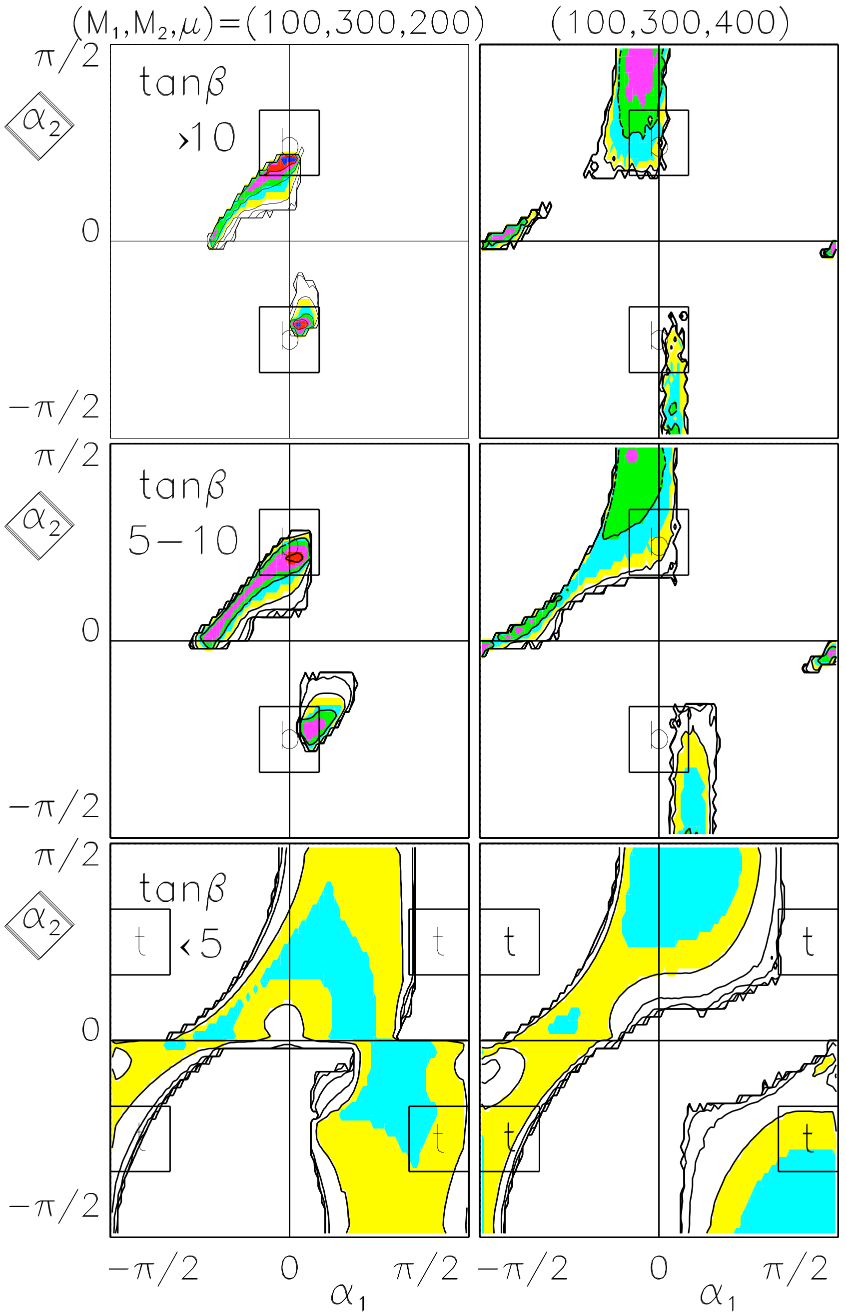
Regions in (α_1, α_2) populated by allowed (by unitarity) solutions

M_1, M_2=100,300 GeV

μ=200, 400 GeV

3 slices of tanβ





Experimental Bounds depending on Neutral Sector

- Choose $M_1 \leq M_2,\ \mu^2$
- Loop over $\tan\beta,\ M_{H^\pm}$
- Scan over $\hat{\boldsymbol{\alpha}} \in \boldsymbol{\alpha}_+ \in \boldsymbol{\alpha}$
- Form χ^2
- Select point in $\hat{\boldsymbol{\alpha}}$ with lowest χ^2

(try to be as generous as possible)

M$_1$, M$_2$, M$_3$=

100, 300, 500 GeV

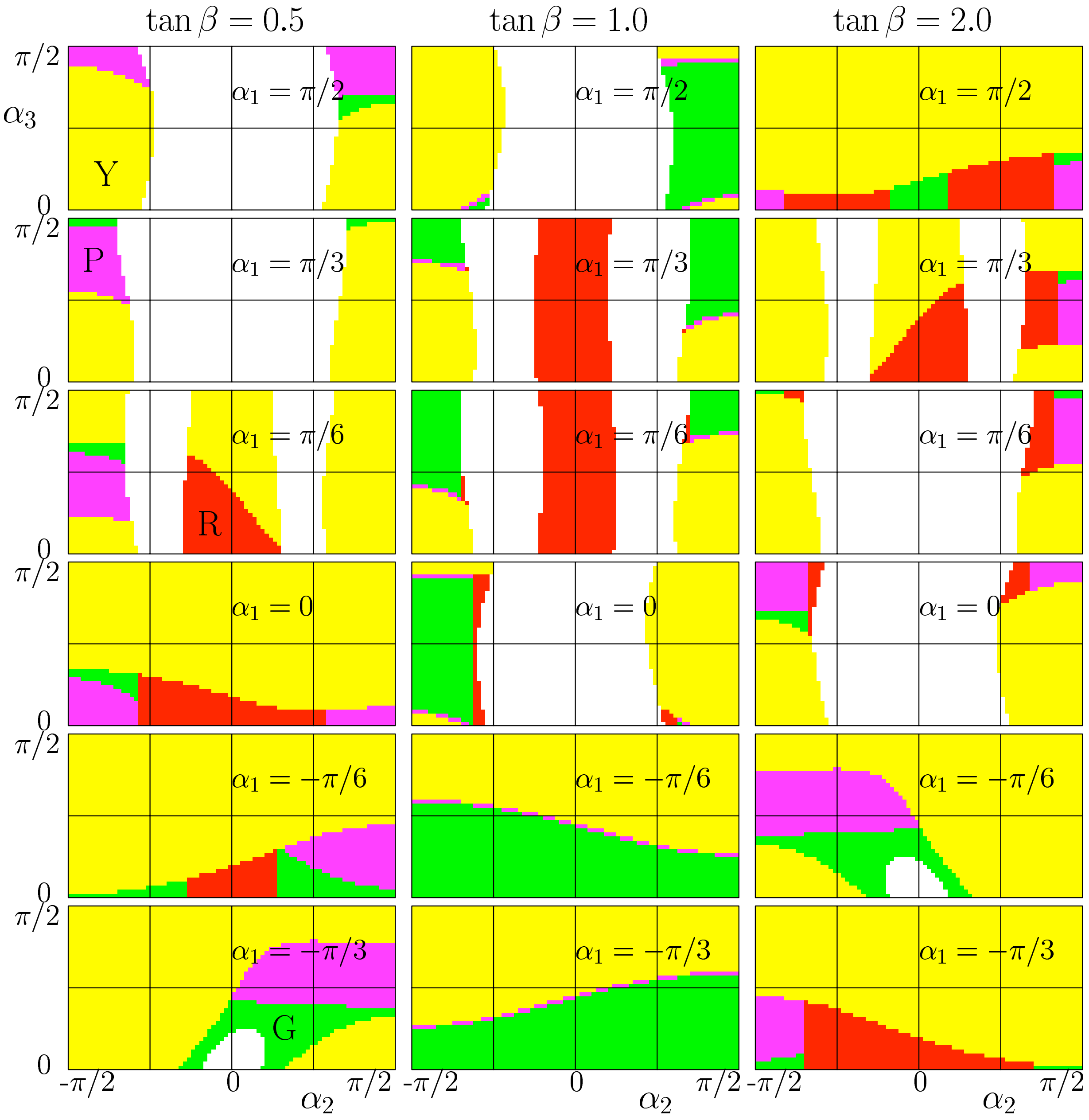
White: positivity violated

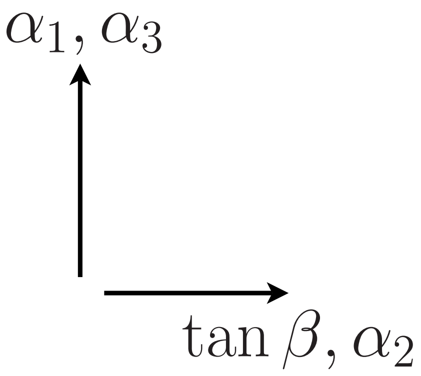
Yellow: unitarity violated

Red: LEP2 search violated

Purple: $\Delta\rho$ violated

Green: OK





- For fixed $\tan\beta$ and $M_{H^\pm}$ take

$$\hat{\chi}_i^2 = \min_{\hat{\boldsymbol{\alpha}} \in \boldsymbol{\alpha}_+} \chi_i^2$$

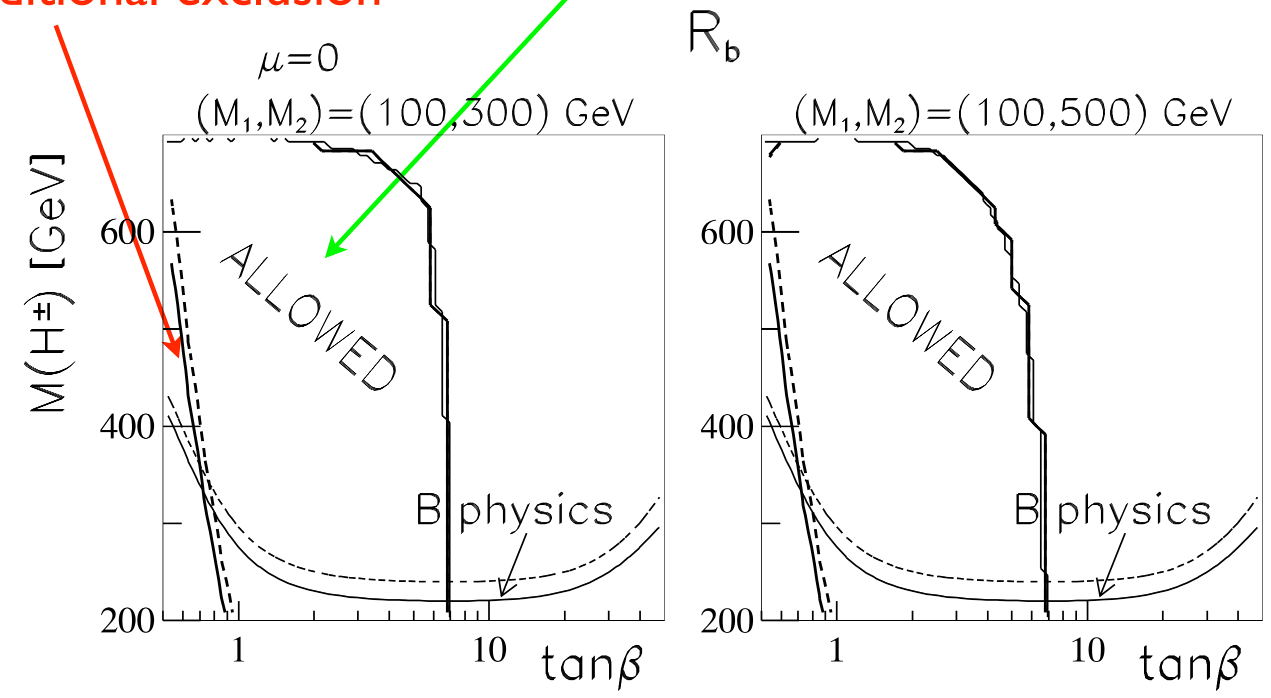
  where χ_i^2 is minimized over the part $\hat{\boldsymbol{\alpha}}$ of the $\boldsymbol{\alpha}_+$ space for which positivity and also unitarity are satisfied.

$\Gamma_{Z \to bb}$ or R_b

Additional exclusion

Not excluded

$\mu^2 = 0$



Denner, Guth, Hollik, Kuhn (1991) extended to CP-violating case in El Kaffas et al, hep-ph/ 0605142
(Neutral Higgs exchange only ‘relevant’ at high tanβ)

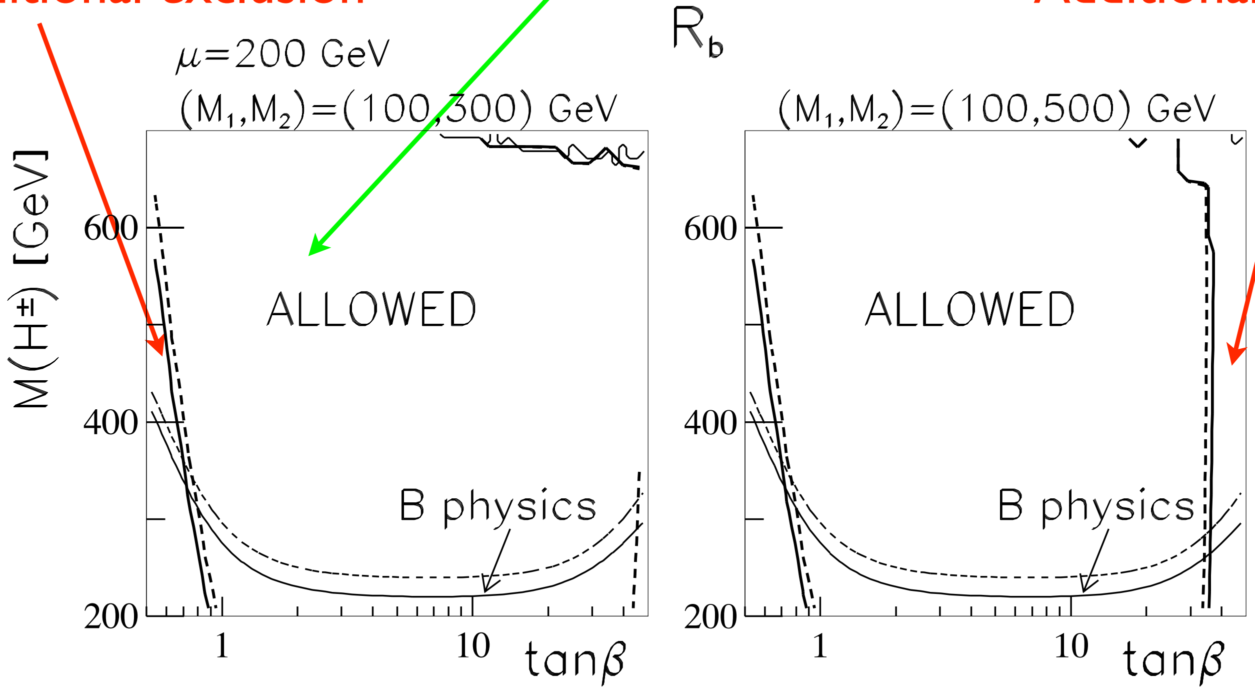
Consider as not excluded: region where some $(\alpha_1, \alpha_2, \alpha_3) \in \hat{\boldsymbol{\alpha}}$ satisfies $\chi^2 < \chi^2(90\%, 95\%\mathrm{CL})$

$\Gamma_Z \to bb$ or R_b

Not excluded

Additional exclusion

Additional exclusion

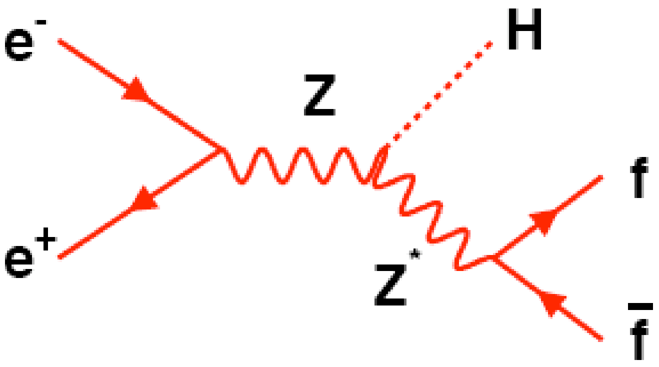


Neutral Higgs exchange
‘relevant’ at high tanβ

$$\mu^2 = (200\ \mathrm{GeV})^2$$

LEP2

Production mechanism at LEP



No SM Higgs particle found up to M=115 GeV

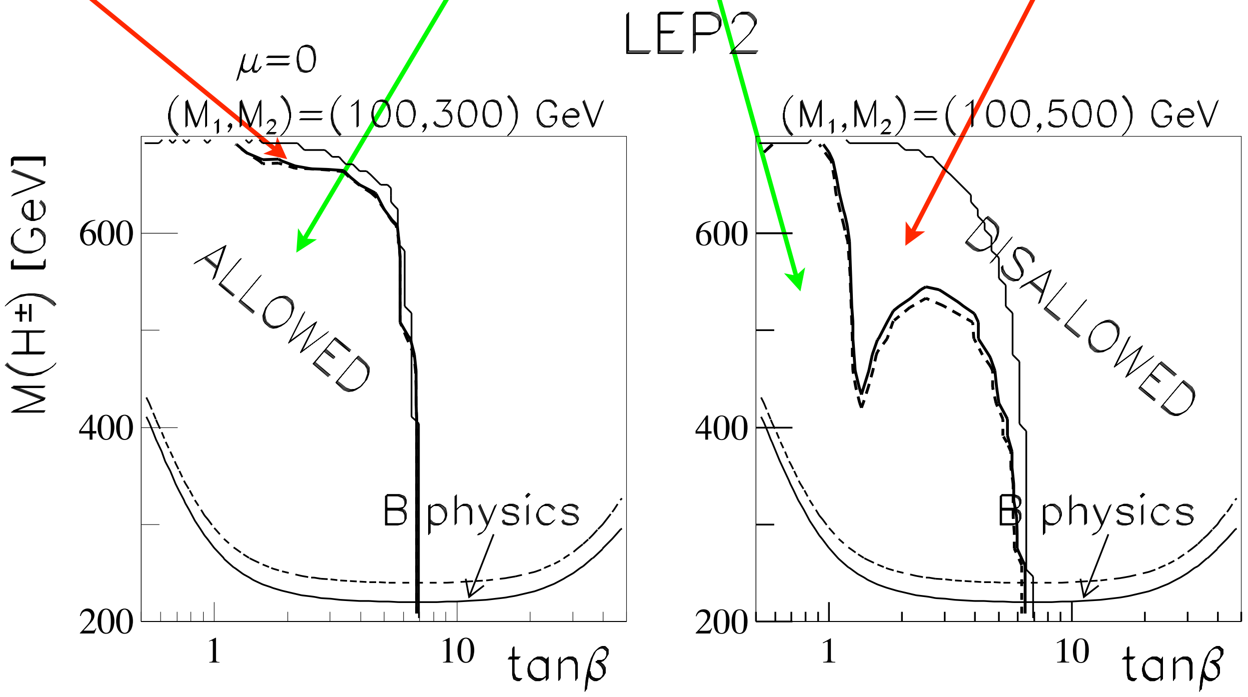
Less exclusion in 2HDM

2HDM: $\sigma_{Z(h\to X)} = \sigma^{\mathrm{ew}}_{Zh} \times C^2_{Z(h\to X)}$

suppressed production: $H_j ZZ: \quad [\cos\beta R_{j1} + \sin\beta R_{j2}], \quad \text{for } j=1$

modified $b\bar{b}$ decay rate: $\dfrac{1}{\cos^2\beta}[R^2_{11} + \sin^2\beta R^2_{13}]$

LEP2 non-discovery



Additional exclusion

Not excluded

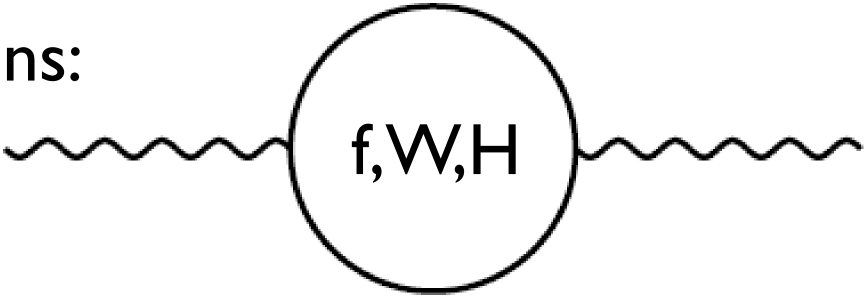
Additional exclusion

$$\mu^2 = 0$$

$\Delta\rho$

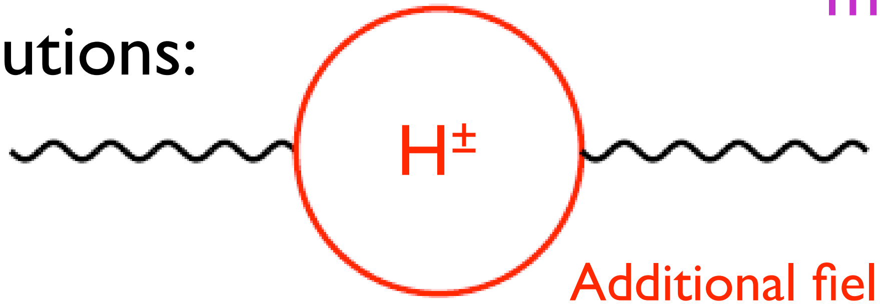
ρ parameter measures difference of self energies of W and Z

SM contributions:



2HDM contributions:

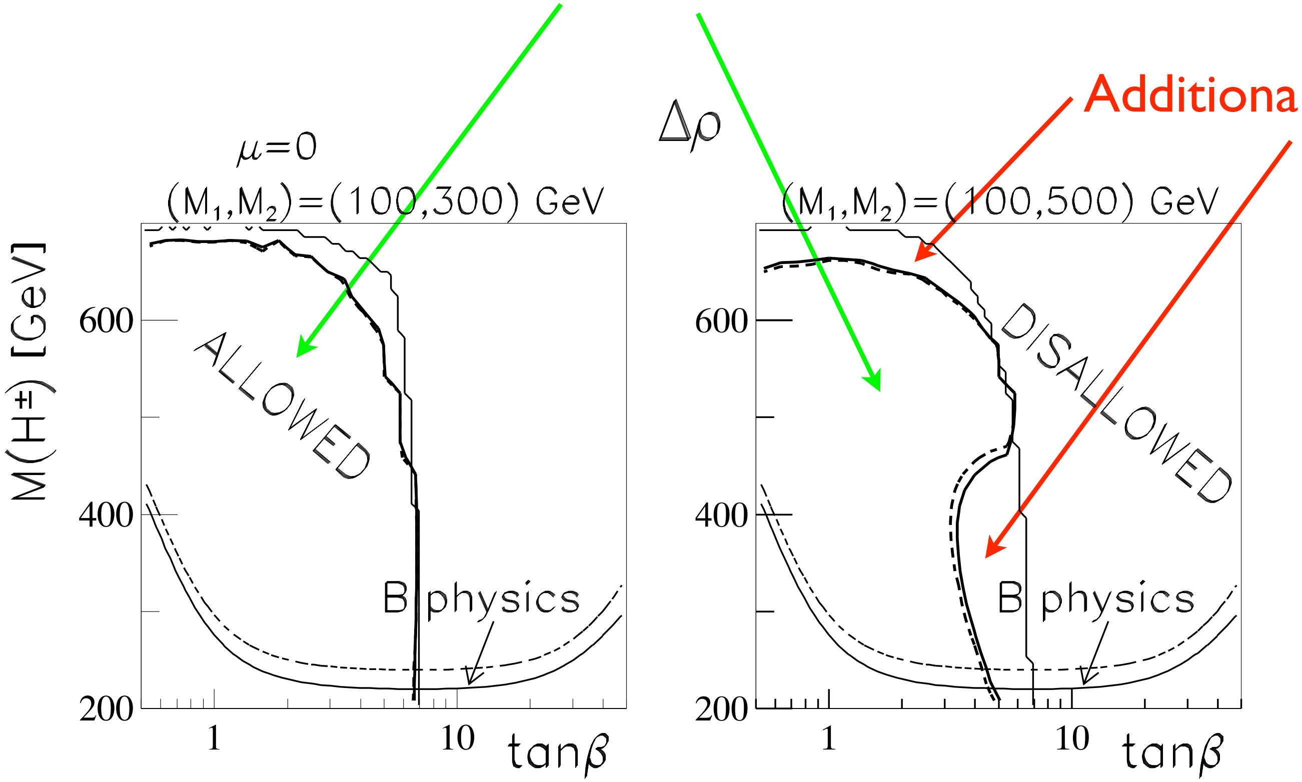
many diagrams



Additional fields must have masses "close" to M_W, M_Z and masses not too widely spaced

Δρ

Bertolini (1986)
extended to CP-violating
case in El Kaffas et al,
hep-ph/0605142



Not excluded

Additional exclusion

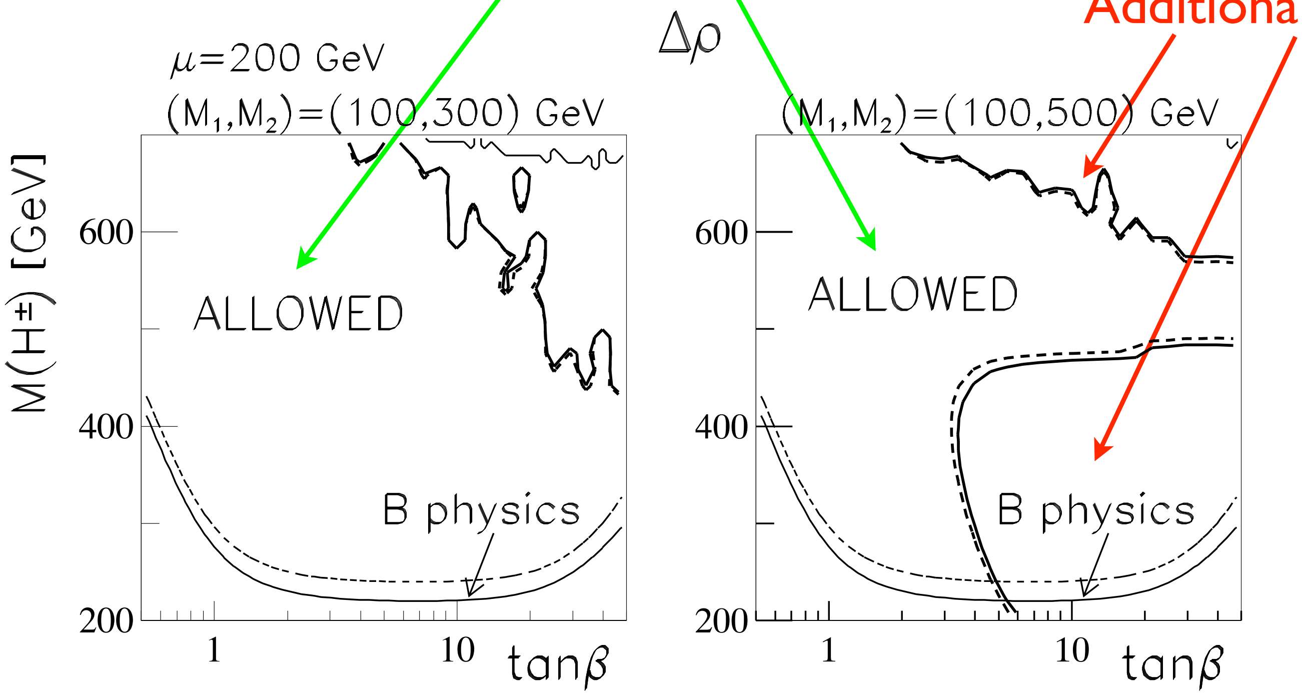
$$\mu^2 = 0$$

Δρ

Bertolini (1986)
extended to CP-violating
case in El Kaffas et al,
hep-ph/0605142

Not excluded

Additional exclusion



$$\mu^2 = (200\ \mathrm{GeV})^2$$

$\Delta\rho$ contributions $\tan\beta \gg 1$

$$A^{HH}_{WW}(0) - \cos^2\theta_W\, A^{HH}_{ZZ}(0)$$

$$\to \frac{g^2}{64\pi^2}\sum_j \Big[(R^2_{j1} + R^2_{j3}) F_{\Delta\rho}(M^2_{H^\pm}, M^2_j)$$

$$- \sum_{k>j} (R_{j1}R_{k3} - R_{k1}R_{j3})^2 F_{\Delta\rho}(M^2_j, M^2_k)\Big]$$

No penalty for $M_2 \simeq M_3 \simeq M_{H^\pm}$ (cancellations)

Unimportant $M_W \approx M_Z$

$$A^{HG}_{WW}(0) - \cos^2\theta_W\, A^{HG}_{ZZ}(0)$$

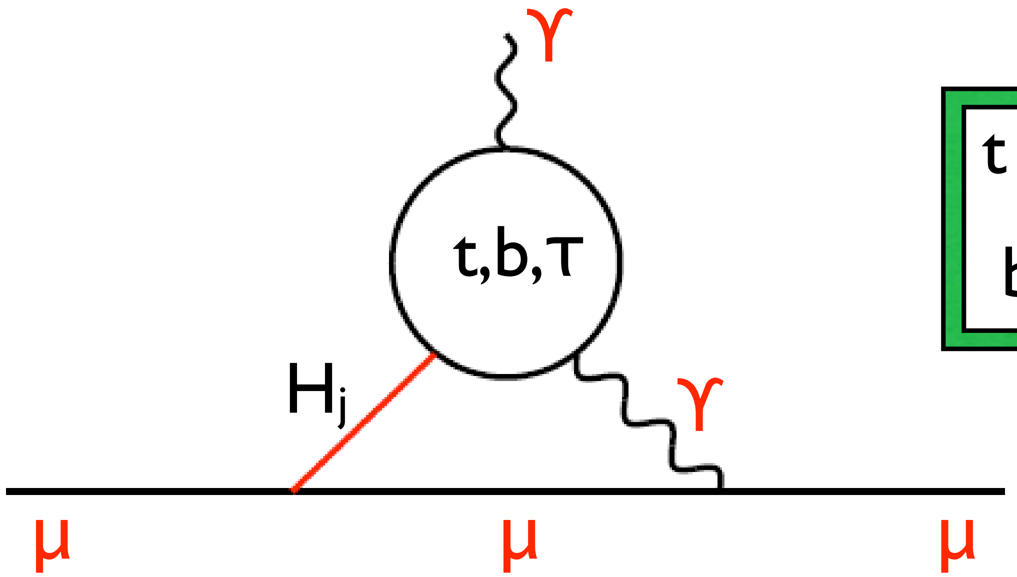
$$\to \frac{g^2}{64\pi^2}\Big[\sum_j R^2_{j2}\Big(3F_{\Delta\rho}(M^2_Z, M^2_j) - 3F_{\Delta\rho}(M^2_W, M^2_j)\Big)$$

$$+ 3F_{\Delta\rho}(M^2_W, M^2_0) - 3F_{\Delta\rho}(M^2_Z, M^2_0)\Big]$$

$$F_{\Delta\rho}(m^2_1, m^2_2) = \tfrac{1}{2}(m^2_1 + m^2_2) - \frac{m^2_1 m^2_2}{m^2_1 - m^2_2}\log\frac{m^2_1}{m^2_2}$$

$(g\text{-}2)_\mu$

Two-loop Barr-Zee effect

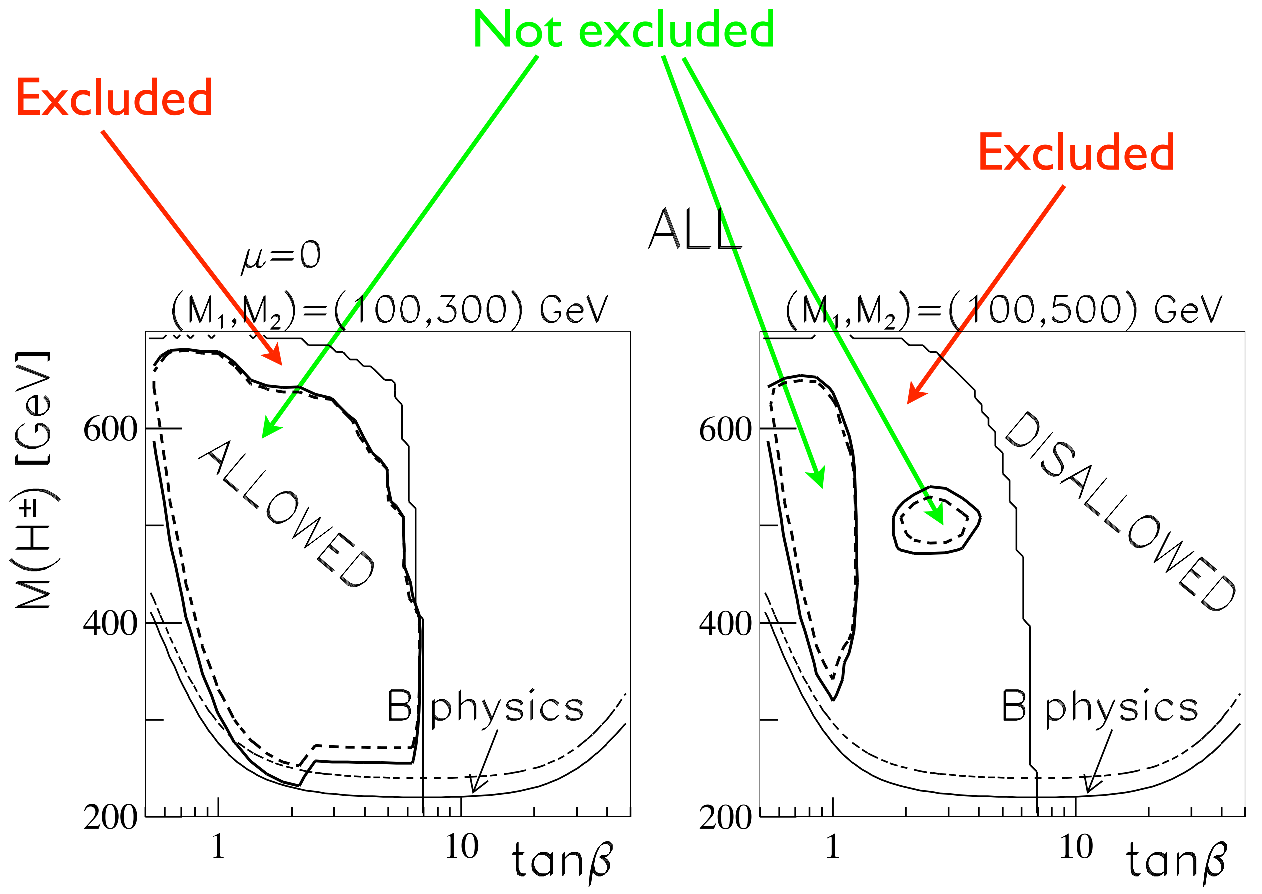


t	at low tanβ
b,τ	at high tanβ

may produce effect comparable with (very high) precision

Only relevant at very high tanβ and/or low $M_{H^\pm}$

Combine all constraints

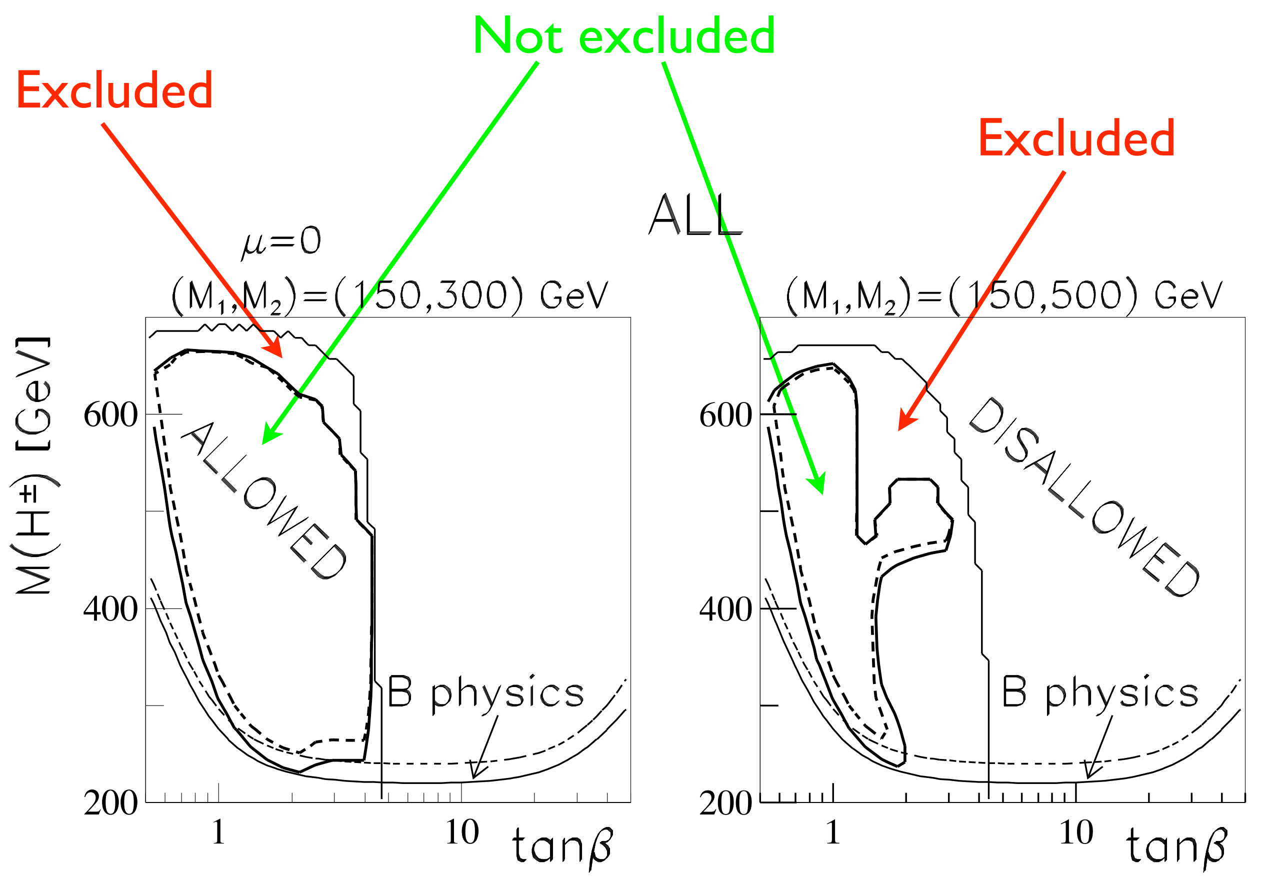


$(M_1,M_2)=(100, 300)$ GeV

$(M_1,M_2)=(100, 500)$ GeV

$$\mu^2 = 0$$

Combine all constraints

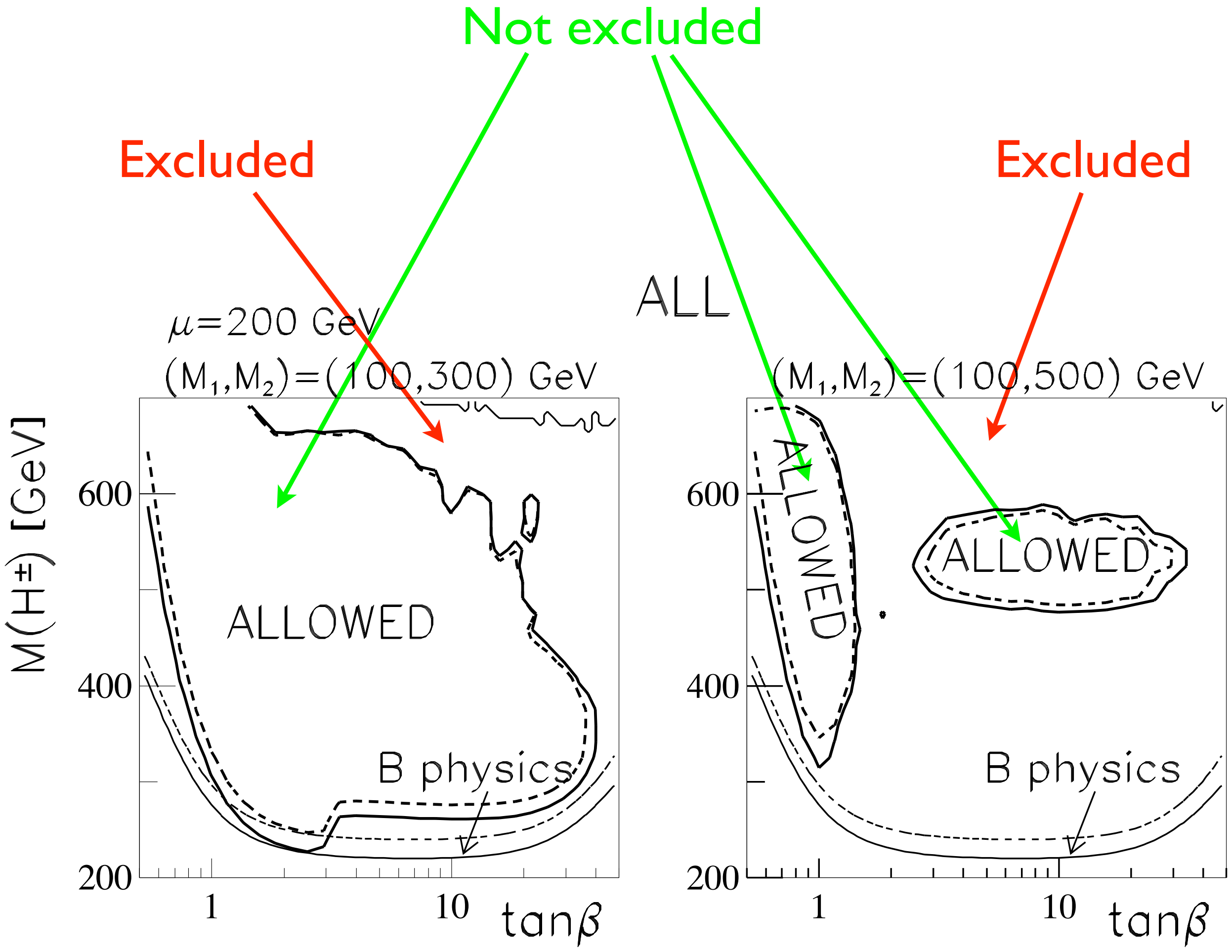


(M1,M2)=(150, 300) GeV

(M1,M2)=(150, 500) GeV

$\mu^2 = 0$

Combine all constraints



Not excluded

Excluded

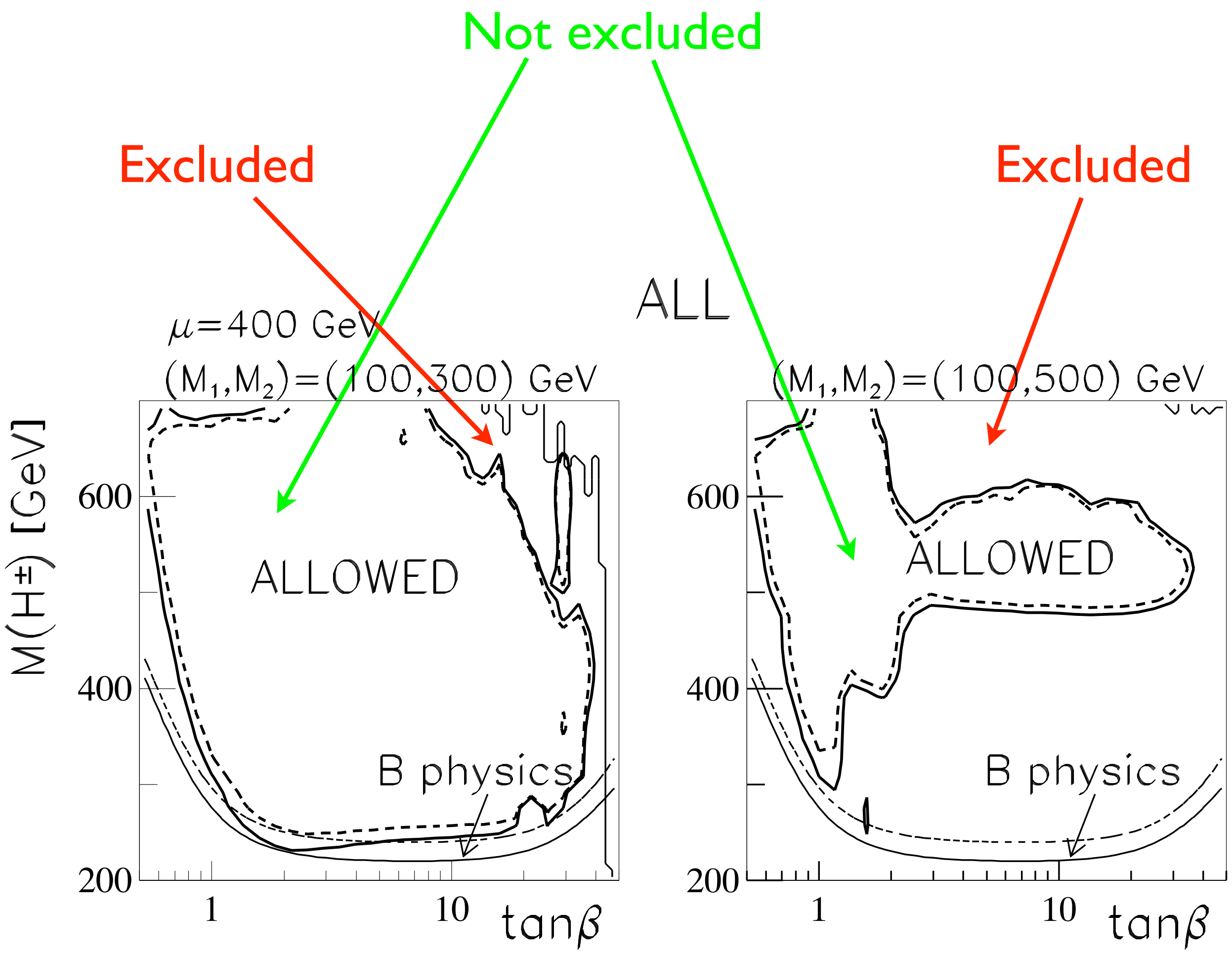
Excluded

(M$_1$,M$_2$)=(100, 300) GeV

(M$_1$,M$_2$)=(100, 500) GeV

$$\mu^2 = (200 \text{ GeV})^2$$

Combine all constraints

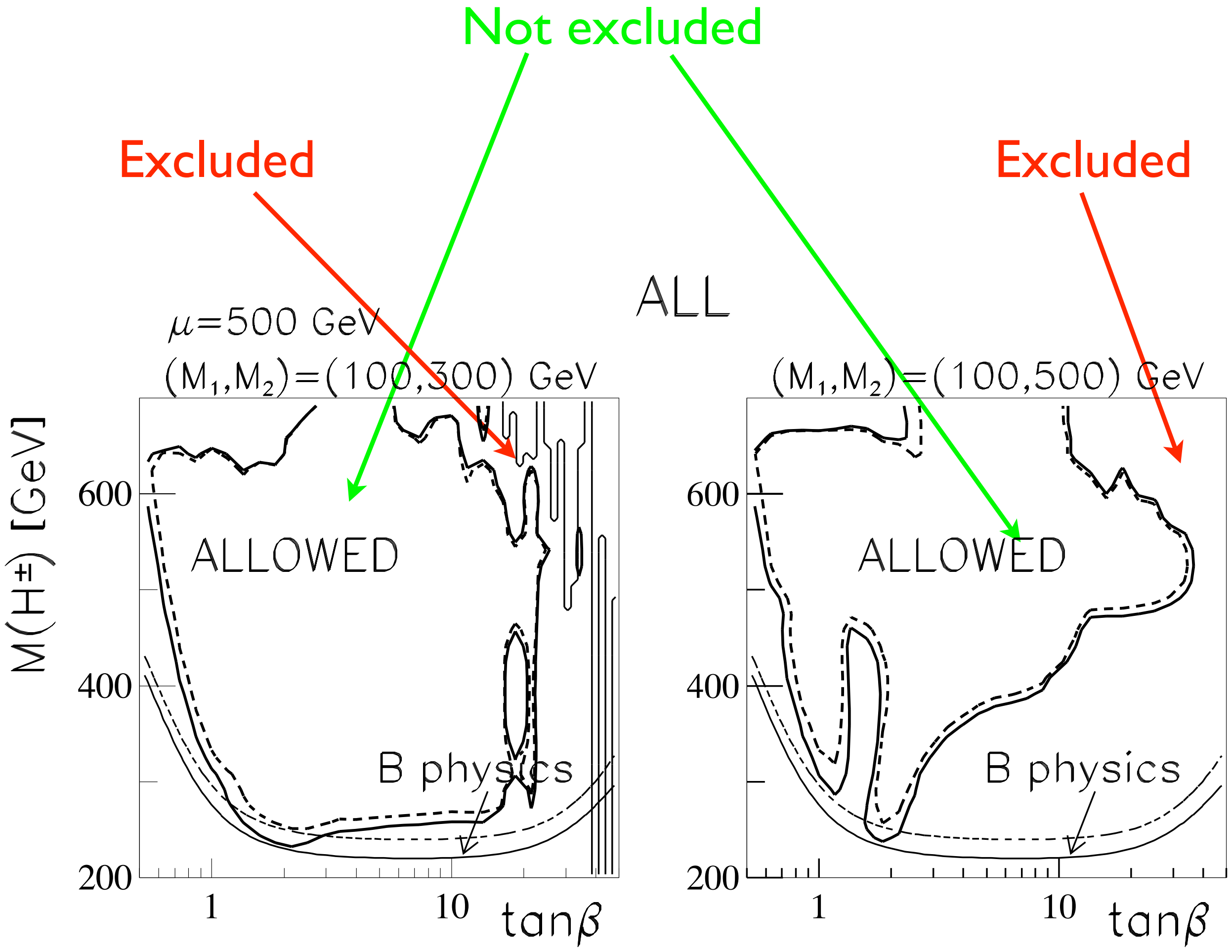


(M_1,M_2)=(100, 300) GeV

(M_1,M_2)=(100, 500) GeV

$$\mu^2 = (400\ \mathrm{GeV})^2$$

Combine all constraints



Not excluded

Excluded

Excluded

(M$_1$,M$_2$)=(100, 300) GeV

(M$_1$,M$_2$)=(100, 500) GeV

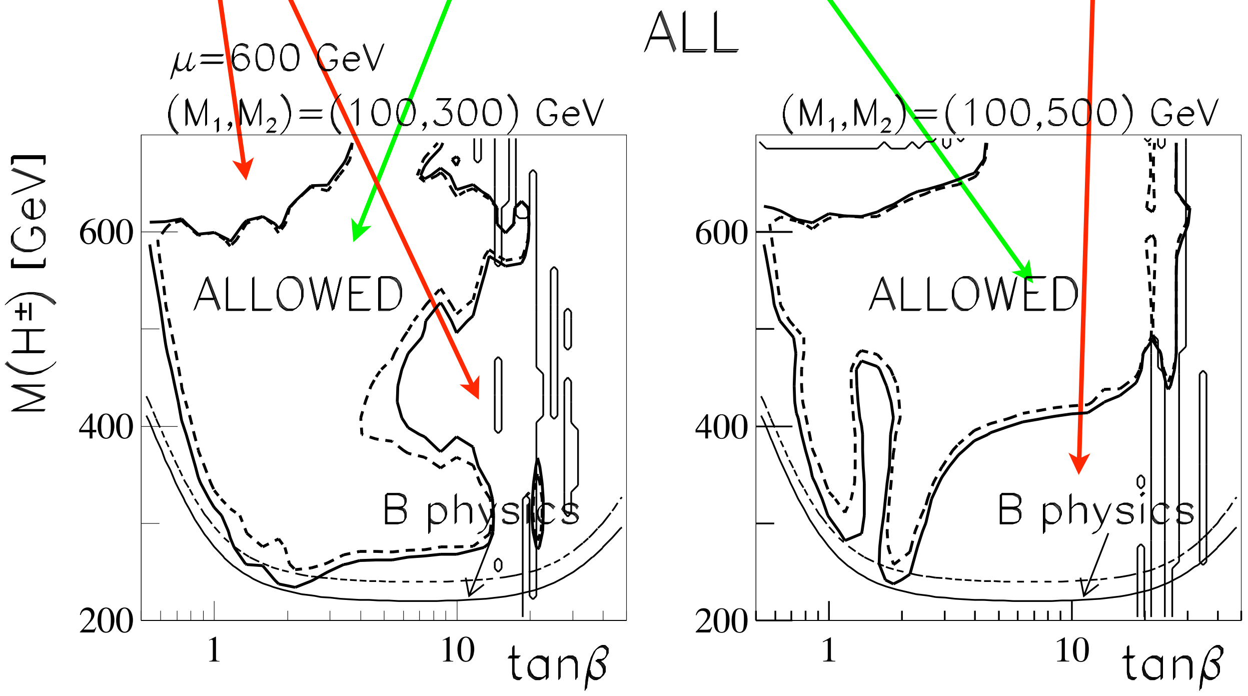
$$\mu^2 = (500\ \mathrm{GeV})^2$$

Combine all constraints

Not excluded

Excluded

Excluded

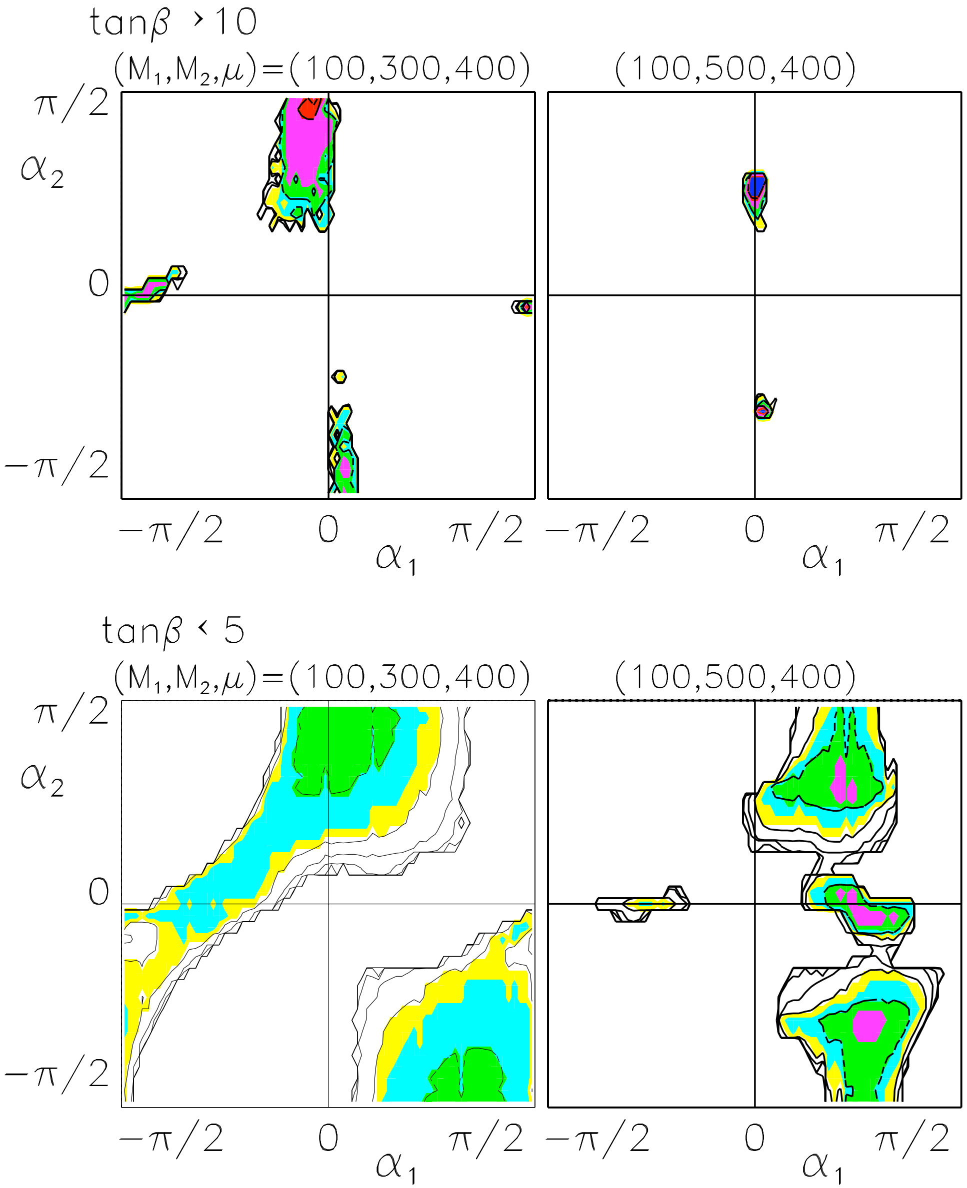


(M$_1$,M$_2$)=(100, 300) GeV

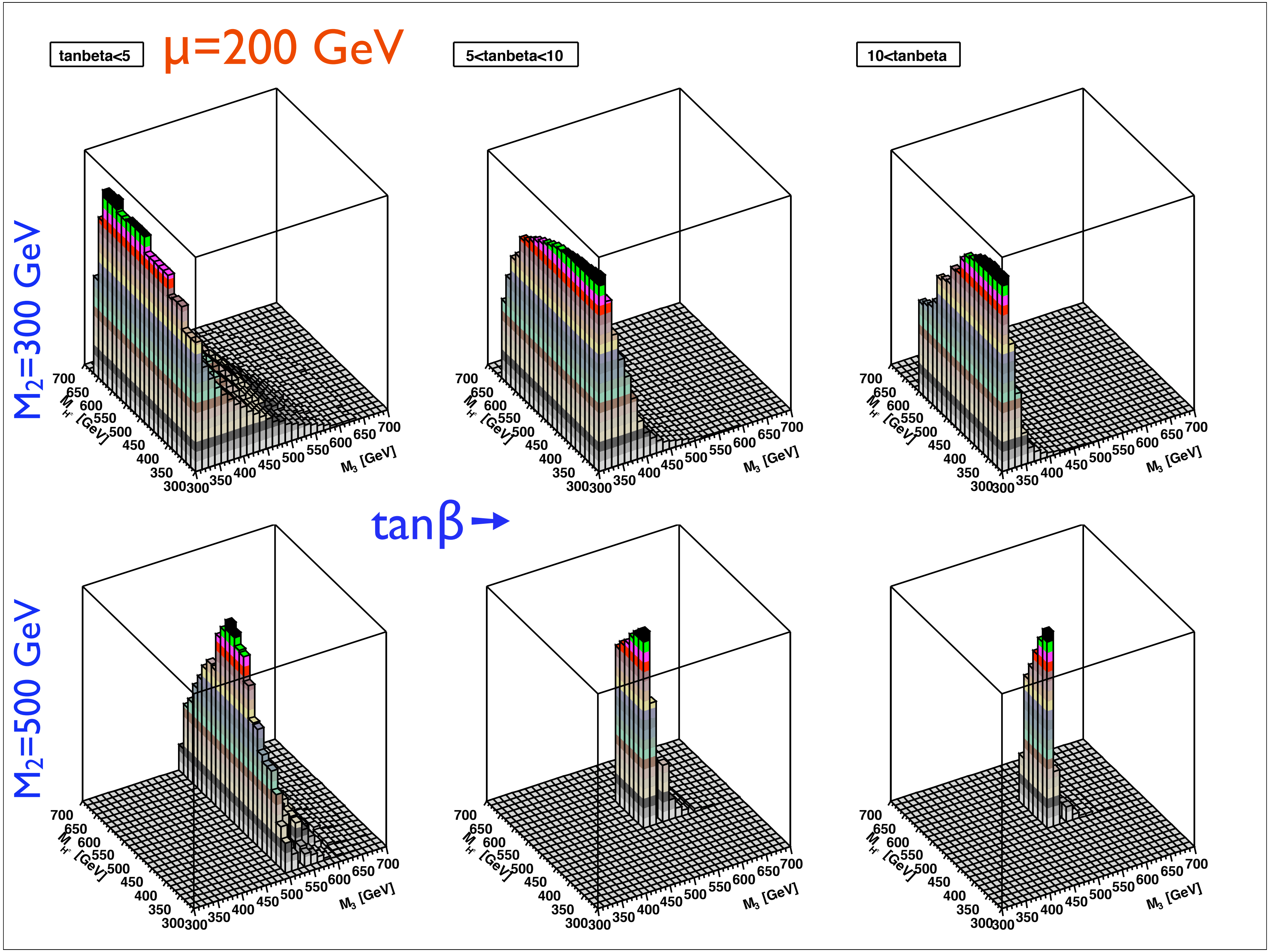
(M$_1$,M$_2$)=(100, 500) GeV

$$\mu^2 = (600\ \mathrm{GeV})^2$$

Profile of surviving parameter space



How is M_3 distributed?



μ=200 GeV

M_2=300 GeV

M_2=500 GeV

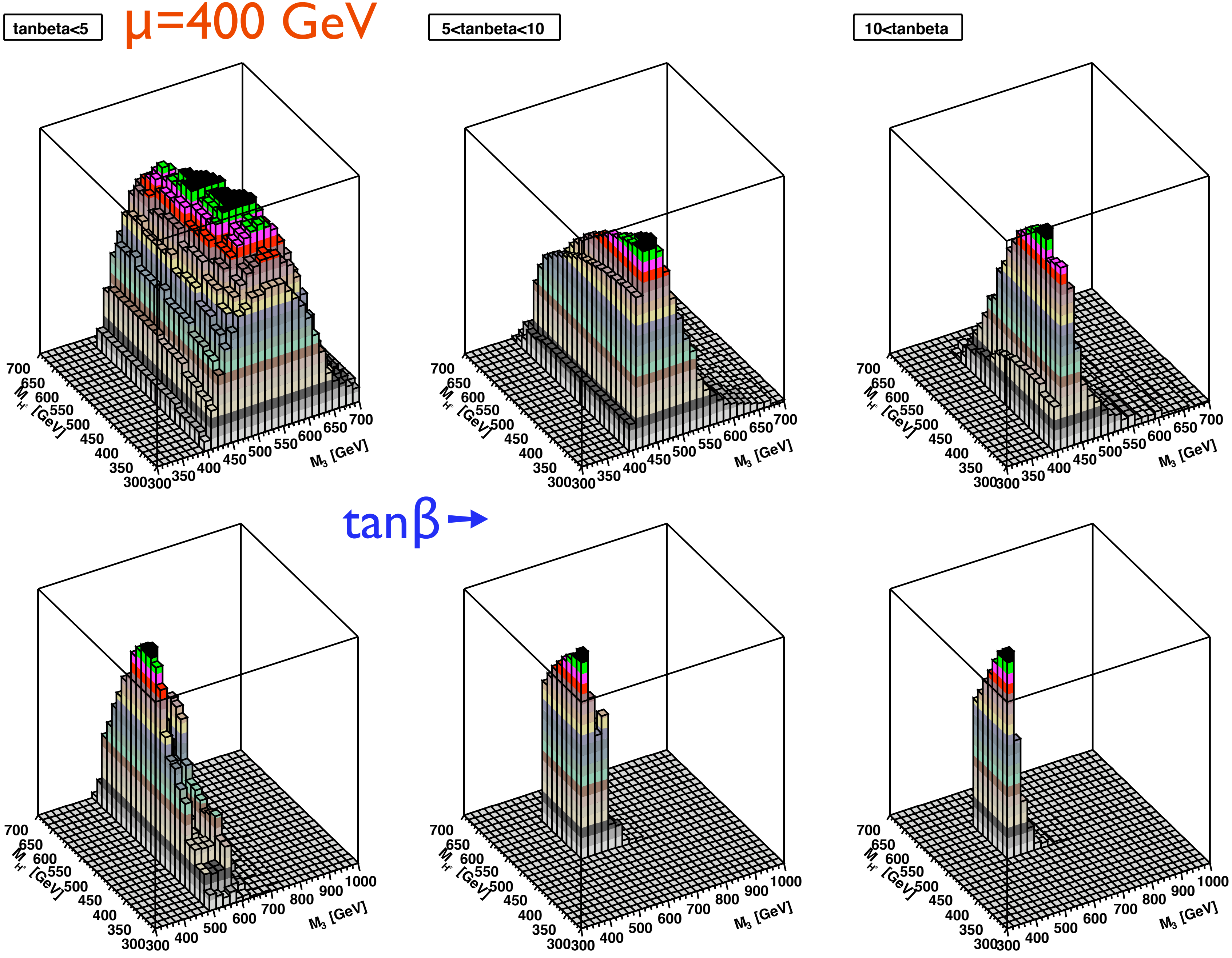
tanβ →

μ=400 GeV

M_2=300 GeV

M_2=500 GeV

tanβ →



Trilinear couplings

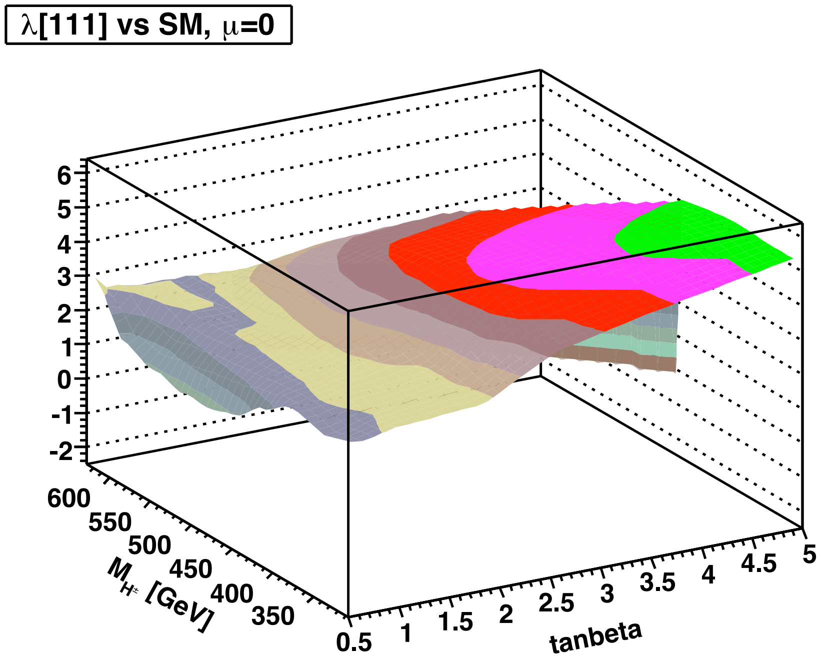
$$\lambda_{ijk} = \sum_{m\leq n\leq o=1,2,3}^{*} R_{i'm}R_{j'n}R_{k'o}\,\frac{\partial^3 V}{\partial\eta_m\partial\eta_n\partial\eta_o}$$

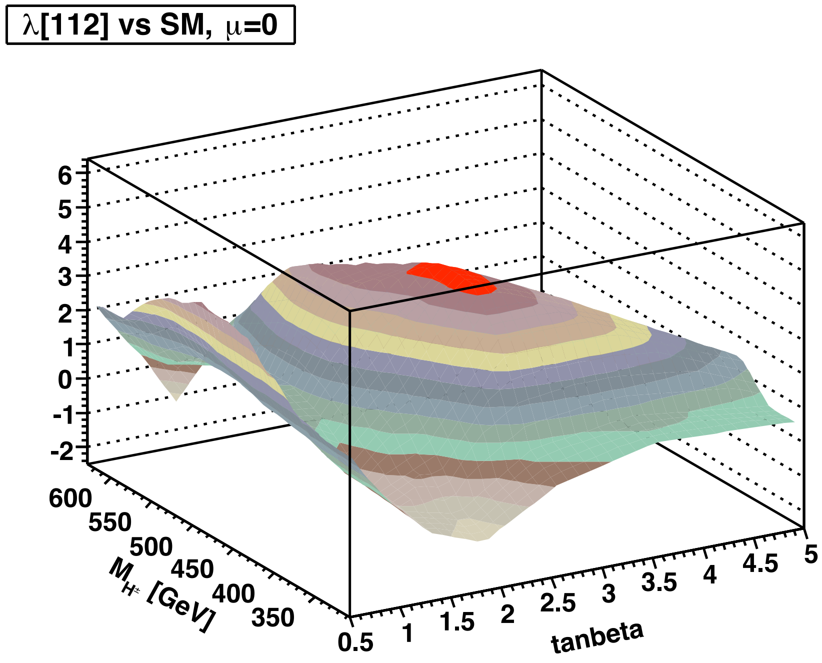
$$= \sum_{m\leq n\leq o=1,2,3}^{*} R_{i'm}R_{j'n}R_{k'o}\,a_{mno}$$

i', j', k' is permutation of i, j, k

Compare with SM coupling:

$$\lambda_{HHH}^{\rm SM} = \frac{3M_H^2}{v}.$$


λ[111] vs SM, μ=0


λ[112] vs SM, μ=0

Trilinear couplings of 2HDM are typically LARGER than SM coupling

Summary

- B physics data exclude low $M_{H^\pm}$ and low $\tan\beta$
- Unitarity excludes high $\tan\beta$ and high $M_{H^\pm}$
- Neutral sector constraints allow only $\boldsymbol{\alpha}_i \in \hat{\boldsymbol{\alpha}}$

$$i = \{\Delta\Gamma_b, \mathrm{LEP2}, \Delta\rho, (g-2)_\mu\}$$

Do the not-excluded $\boldsymbol{\alpha}_i$ have any overlap?

“Yes”

$(g\text{-}2)_\mu$ irrelevant
$\Delta\rho$ very constraining

LHC may provide total exclusion (or discovery)