Closed Strings in Open String Field Theory

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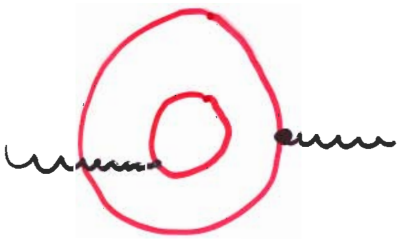
Question: open or closed string more fundamental ?

* closed: how to classify D-branes ?

* open: where are closed string d.o.f. in H_{open} :

at 1-Loop:

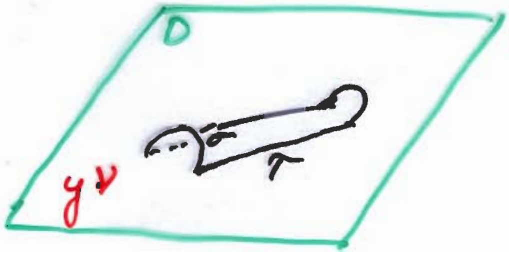


contains closed string poles

Unitarity ??

At tree level: D-branes in bosonic string theory:

* ground state gives rise to tachyon on world volume of D



String field:

creation op.

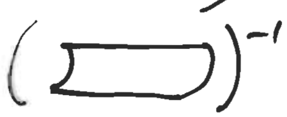
$$\Psi[X^\nu(\sigma,\tau)] = \big(T(y^\nu)\, c_{-1} + A_\mu(y^\nu)\, \alpha^\nu_{-1} c_{-1} + \dots\big)|0\rangle$$

tachyon

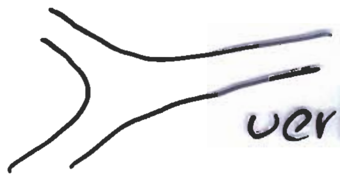
photon

ground state of harm. osc.

$$I[\Psi] = \int \Psi * Q^B \Psi + \underbrace{\Psi * \Psi * \Psi}$$



inv. propagator



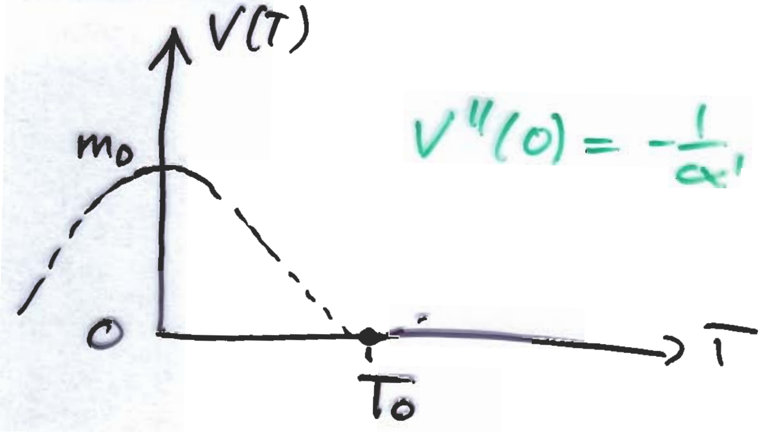
vertex

$\Psi * \Psi \;\hat{=}\;$ OPE on string world sheet.

* class. solution: $Q^B \bar{\Psi} + \bar{\Psi} * \bar{\Psi} = 0$

$$\Rightarrow \Box_{y^\nu} T(y^\nu) + \frac{1}{\alpha'} T(y^\nu) = 0 + \mathcal{O}(T^2; A_\mu, \ldots)$$

* potential for T



* what is the physics at $T = T_0$?

$(Q^B + \bar{\Psi}) =: \tilde{Q}$ has trivial cohomology (M. Schnabl '05)

$\Longrightarrow$ no physical open string excitations

* Analogy: Abelian Higgs model ($\phi = H e^{i\varphi}$

$$L(H, \varphi, A_\nu) = (\partial_\mu H)^2 + (H - H_0)^2 + \underbrace{H^2}_{e^{-T}} (\underbrace{A_\mu}_{B_{\mu\nu}} - \underbrace{\partial_\mu \varphi}_{A_\mu})^2$$

– in the symmetric phase φ is not a good coordinate (but ϕ is!)

Analogy: $e^{i\varphi} \hat{=} \Psi_{\text{open}}[X]$

* Conjecture (Sen '98):

i) $T = T_0$ describes the closed string vacuum.

ii) Excitations at T_0 are closed strings.

* Proposal for OSFT (Lazaroiu, ...):

$(\Psi, *, Q)$ is a differential graded algebra (DG)

Equivalence classes of deformations of $(\Psi, *, Q)$ are given by the Hochschild complex.

ex: (i) $B_{\mu\nu} = \text{const} \leftrightarrow$ deformation of $*$ product.

(ii) in top. string theory: Hoch. Coh. $\simeq$ gravitational primaries

* Proposal for BSFT (Shatashvili, Baumgartl, I.S)

~ boundary σ-model with action

$$I = \int_D L_p(X^\mu, G_{\mu\nu}, B_{\mu\nu}) + \oint_{\partial D} B(t^I, X^\mu)$$

$$\left.\frac{\partial B(t^I, X^\mu)}{\partial t^J}\right|_{t^J=0} = V_J(X^\mu)$$

open string vertex operator.

$$Z[t^I] = \int D[X^\mu]\, e^{-I}$$

off-shell action:

β-functions

$$S[t^I] = (\beta^I \partial_I - 1)\, Z[t^I]$$

critical pts.: $dS = 0$

$$\Longleftrightarrow \beta^I = 0$$

Proposition: Let $\mathcal{M} = (G_{\mu\nu}, B_{\mu\nu})$

(massless closed string modes)

Then:

$$S[\{t^I\}, \mathcal{M}] \equiv S[\{t^I\} + \Delta t, \hat{\mathcal{M}}]$$

some (non-local) collective open string excitation

$\therefore$ assumes $\mathcal{M}, \hat{\mathcal{M}}$ on-shell, i.e. $\beta^{(\hat{\mu})} = 0$

i.e. shift in closed string background can be absorbed in collective open string excitation.

$\therefore$ $T_{OSFT} = T_{OSFT}(T, A_\mu, \dots)$

formally $\Psi[X] \sim e^{\oint_{\partial D} B[X, t^I]}$

* key observation (proven for group manifolds)

$$\boxed{Z_{\mathcal{M}}[t^I] = Z^{\text{bulk}}_{\mathcal{M}} \cdot Z^{\text{bdy}}_{\hat{\mathcal{M}}}[t^I + \Delta t]}$$

$$Z^{\text{bulk}}_{\mathcal{M}} = \int_{X_0|_{\partial D}=0} D[X_0]\, e^{-I(X_0,\mathcal{M})} \quad ; \; t^I\text{-indep.}$$

Normalization

$$Z^{\text{bdy}}_{\hat{\mathcal{M}}} = \int d\mu_{\hat{\mathcal{M}}}[Y(\theta)]\; e^{-A[Y] - \oint_{\partial D} B(Y, t^I)}$$

uniquely determined by boundary data.

$Y(\theta)$: boundary field : $X = X_0 + Y$

Examples:

(i) $\mathcal{M} = (\mathbb{R}^n, \delta_{\mu\nu}, B_{\mu\nu} = \text{const})$

$\hat{\mathcal{M}} = (\mathbb{R}^n, \delta_{\mu\nu}, B_{\mu\nu} = 0)$

$$X^\mu(z,\bar z) = X_0^\mu(z,\bar z) + Y^\mu(z,\bar z) \; ; \; \Delta Y^\mu(z,\bar z) = 0$$

$$\Rightarrow \; Y^\mu(z,\bar z) = f^\mu(z) + \bar f^\mu(\bar z).$$

7.1

$$A[y^\mu] = \oint_{\partial D} \left\{ f^\mu(\theta)\partial_\theta \bar{f}_\mu(\theta) + \underbrace{y^\mu(\theta)\partial_\theta y^\nu(\theta)\, B_{\mu\nu}}_{=\Delta t \text{ (local)}} \right\}$$

here $\Delta t \sim$ * product

(ii) $B_{\mu\nu} \neq$ constant :

- $\mathcal{M} = (S^3 \simeq SU(2),\ dB = vol(S^3))$
- $\beta_\mu = 0$ ✓

$$SU(2)^{\mathbb{C}} \ni g(z,\bar{z}) = g_0(z,\bar{z}) \underbrace{k(z,\bar{z})}_{= h(z)\,\bar{h}(\bar{z})}$$

$\Rightarrow A[h,\bar{h}]$ is a group 2-cocycle :

$$1 \longrightarrow U(1) \longrightarrow \widehat{LG} \longrightarrow LG \longrightarrow 1$$

with:

$$(\delta A)(a,b,c) = A[a,bc] + A[b,c] - A[ab,c] - A[a,b] = 0$$

$$A[a,b] \sim A[a,b] + \underbrace{\delta B(a,b)}_{= B(ab)} \hat{=} \text{ open string d.o.f.}$$

$\therefore$ for given A $\exists B$ s.t. $dS = 0$ (conformal)

$\therefore$ in ex. (i) $A[f,\bar{f}] = \delta B(f,\bar{f}) = B(f+\bar{f}=y)$

$\therefore$ $A[h,\bar{h}]$ can generically not be written as a boundary integral but is uniquely determined by the boundary data $h(\theta), \bar{h}(\theta)$

Question: Is there a relation between the "cocycles" $A[f,\bar{f}]$" and the Hochschild Coh of OSFT ?

What can we do with this :

* stability of D-branes in curved space :
  - space filling brane is not on-shell
  - RG-flow to $\begin{cases} \bullet \text{ point-like } D\emptyset \\ \bullet \text{ spherical 2-brane} \end{cases}$

choose coordinates on $G = SU(2)$:

$$h(\theta) = e^{i\lambda f^a(\theta) T^a} \; ; \; \bar{h}(\theta) = e^{-i\lambda \bar{f}^a T_a}$$

$$\to A[h,\bar{h}] = \oint_{\partial D} f^a \partial_\theta \bar{f}^a + i\lambda \underbrace{\varepsilon^{abc} f^a(\theta) \bar{f}^b(\theta) \partial_\theta f^c(\theta)}_{\Delta t_{\text{non-local}}} + \text{h.c.} + O(\lambda^2)$$

$\therefore \lambda = 0$: BSFT in flat space

measure : $d\mu(h,\bar{h}) = D[f] D[\bar{f}] \, e^{\underbrace{-\lambda^2}_{\Delta t_{\text{local}}} \int_{\partial D} f^a \bar{f}^a + O(\lambda^4)}$

$\Delta t_{\text{local}} \hat{=}$ open string tachyon

$$T(y^a) = u \, y^a y^a \quad \Leftarrow \text{open string tachyon}$$

$$u = \lambda^2$$

$\beta_u = -u(1-\lambda^2) \implies$ tachyon condensation on space-filling brane

End point of flow ?

- $u = \infty \;\hat{=}\;$ pt-like D$\emptyset$ brane ✓

- spherical D2 brane ? radius

$$\to \text{add } B[t^I] = \overset{\mathbb{R}}{\varsigma} \int_{\partial D} (y^2 - \overset{\text{radius}}{c^2})$$

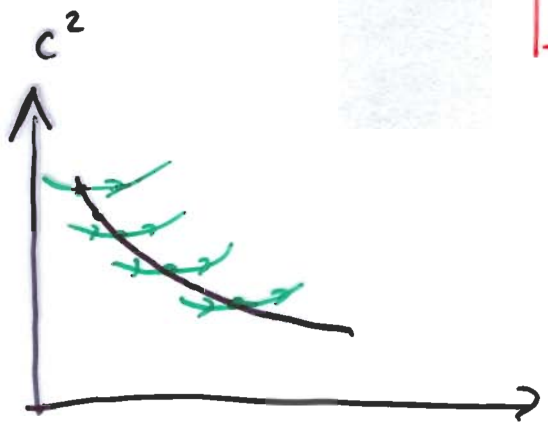
* Analyze RG flow for

$$\oint_{\partial D} B(y) = g \oint (y^2 - c^2)^2$$

<u>Result</u>: $\beta_g < 0$

$$\beta_{c^2} = 1 - 2c^2\lambda^2 + O(\lambda^4)$$

$$= 0 \quad \boxed{\text{for } c^2 = \frac{1}{2\lambda^2}}$$



flow towards a spherical 2 brane ?

<u>check</u>: impose constraint:

$$Z^{bdy}_{\alpha^2}[t^I] = \int d\mu(y)\, \delta(y^2 - c^2)\, |J|\, e^{-\alpha_2 (f, f) + \oint_{\partial D} B}$$

Result: • $u = A(c^2, \lambda)\, \Lambda^{\text{cut off}}$

$$A(c^2; \lambda) = 0 \text{ for } c^2 \cong \frac{28}{\lambda^2}$$

→ sperical 2-brane is a solution.

Discussion:

* closed string backgrounds appear to be contained in H_{open} as collective excitations.

* including these open strings, string field theory may well be complete and unitary