BW2007 WORKSHOP

III Southeastern European Workshop

"Challenges Beyond the Standard Model"

Renormalizability and Phenomenology of θ-expanded Noncommutative Gauge Field Theory

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Introduction

Example of noncommutativity (NC): Heisenberg algebra

$$[\hat{x}^\mu, p^\nu] = i\hbar\delta^{\mu\nu}, \quad [p^\mu, p^\nu] = 0$$

Constructing models on noncommutative space-time
Motivations: String Theory
Quantum Gravity
Lorentz invariance breaking
Heuristic

* The star product: $[x^\mu, x^\nu]_\star = x^\mu \star x^\nu - x^\nu \star x^\mu = ih\theta^{\mu\nu}$

$$(f \star g)(x) = e^{-\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x}$$

* Noncommutative space is flat Minkowski space:

$$x^\mu \to \hat{x}^\mu \Rightarrow [\hat{x}^\mu, \hat{x}^\nu] = ih\theta^{\mu\nu},$$

θ - constant, antisymmetric and real 4×4 matrix
$h = 1/\Lambda^2_{\mathrm{NC}}$ - NC deformation parameter

* Symmetry extended to enveloping algebra
* Seiberg-Witten map (SW)

There are 2 essential points in which NCGFT differ from standard gauge theories:

* The breakdown of Lorentz invariance with respect to a fixed $\neq 0$ background field $\theta^{\mu\nu}$ (which fixes preferred directions)
* The appearance of new interactions and the modification of standard ones. For exapmle, triple–neutral–gauge boson, 2 fermion–2 gauge bosons, photon-neutrino, etc.

Both properties have a common origin and appear in a number of phenomena
AT VERY HIGH ENERGIES AND/OR VERY SHORT DISTANCES.

CONSTRUCTION

VIA SEIBERG-WITTEN MAP

[N. Seiberg and E. Witten; String theory and non-commutative geometry, JHEP **9909**, 032 (1999)]

[J. Madore, S. Schraml, P. Schupp and J. Wess; Gauge theory on noncommutative spaces, Eur. Phys. J. C**16** (2000) 161]

[B. Jurčo, S. Schraml, P. Schupp and J. Wess; Enveloping algebra valued gauge transformations for non-Abelian gauge groups on non-commutative spaces, Eur. Phys. J. C **17**, 521 (2000)

[X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgenannt; The standard model on non-commutative space-time, EPJ C**23** (2002) 363]

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\to$ gamma gamma, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

[B. Melić, K. Passek-Kumerički, J.T., P. Schupp and M. Wohlgenannt; The Standard Model on Non-Commutative Space-Time: Electroweak Currents and Higgs Sector, EPJ C**24** (2005) 483 ibid. 499]

[F. Brandt, C.P. Martin and F. Ruiz Ruiz; Anomaly freedom in Seiberg-Witten noncommutative gauge theories JHEP **07** (2003) 068]

[M. Buric, D. Latas and V. Radovanovic, Renormalizability of noncommutative SU(N) gauge theory; JHEP **0602** (2006) 046]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; JHEP**03** (2007) 030]

[D. Latas, V. Radovanovic and J.T., Non-commutative SU(N) gauge theories and asymptotic freedom; hep-th/0703018, to appear in PRD.]

* Models based on the Seiberg-Witten mapping
* Expansion in power series in θ $\to$ new vertices
* Any gauge groups
* Arbitrary matter representation
* No charge quantization problem
* No UV/IR mixing due to θ expansion
* Unitarity is OK for: $\theta^{ij} \neq$, $\theta^{0i} = 0$;

carefull cannonical quantization produces always unitary theory: (*Bahns, Fredenhagen, Doplicher, Piaticelli: Time in S matrix treated in form of slices*)

* By covariant generalization of $\theta^{0i} = 0$ to:

$$\theta_{\mu\nu}\theta^{\mu\nu} \;=\; -\theta^2 = \frac{2}{\Lambda^4_{\rm NC}}\left(\vec{B}^2_\theta - \vec{E}^2_\theta\right) > 0$$

known as *perturbative unitarity condition* one avoids potential difficulties with unitarity in noncommutative gauge field theories

* Construction of covariant NCSM Yukawa couplings OK
* One-loop renormalizable gauge sector at the first order in noncommutative parameter θ
* Models 1 & 2: mNCSM & nmNCSM constructed as an effective, anomaly free and partly renormalizable theories
* Model 3: SU(N) GFT constructed as an renormalizable theory via renormalization of $\theta \to$ RGE for noncommutative deformation parameter h

NC gauge transformation

Consider infinitesimal NC local gauge transformation $\widehat{\delta}$ of a fundamental matter field that carries a representation ρ_Ψ

$$\widehat{\delta}\widehat{\Psi} = i\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}$$

In Abelian case ρ_Ψ fixed by the hypercharge.

Covariant coordinates in NC theory introduced in analogy to covariant derivatives in ordinary theory

$$\widehat{x}^\mu = x^\mu + \theta^{\mu\nu}\widehat{A}_\nu$$

Locality

A $\star -$ product of ordinary functions f, g, determined by a Poisson tensor $\theta^{\mu\nu}(x)$, is local function of f, g with finite number of derivatives at each order in θ:

$$f \star g = f \cdot g + \frac{i}{2}\theta^{\mu\nu}(x)\partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2)$$

Gauge equivalence, and consistency conditions

Ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu]$ and $\delta\Psi = i\Lambda \cdot \Psi$ induce non-commutative gauge transformations of the fields $\widehat{A}$, $\widehat{\Psi}$ with gauge parameter $\widehat{\Lambda}$

$$\delta\widehat{A}_\mu = \widehat{\delta}\widehat{A}_\mu \qquad \delta\widehat{\Psi} = \widehat{\delta}\widehat{\Psi}$$

Consistency require that any pair of non-commutative gauge parameters $\widehat{\Lambda}$, $\widehat{\Lambda'}$ satisfy

$$[\widehat{\Lambda}, \widehat{\Lambda'}] + i\delta_\Lambda \widehat{\Lambda'} - i\delta_{\Lambda'}\widehat{\Lambda} = \widehat{[\Lambda, \Lambda']}.$$

Enveloping algebra-valued gauge transformation

The commutator

$$[\widehat{\Lambda}, \widehat{\Lambda}'] = \frac{1}{2}\{\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)\}[T^a, T^b] + \frac{1}{2}[\Lambda_a(x), \Lambda'_b(x)]\{T^a, T^b\}$$

of two Lie algebra-valued NC gauge parameters $\widehat{\Lambda} = \Lambda_a(x)T^a$ and $\widehat{\Lambda}' = \Lambda'_a(x)T^a$ does not close in Lie. For NC SU(N) & Lie algebra traceless condition incompatible with commutator. Extension to enveloping algebra-valued NC gauge parameters and fields.

$$\widehat{\Lambda} = \Lambda^0_a(x)T^a + \Lambda^1_{ab}(x) : T^aT^b : +\Lambda^2_{abc}(x) : T^aT^bT^c : + \ldots$$

Clousing condition for gauge transformation algebra are homogenos differential equations which are solved by iteration, order by order in θ. This solution is known as Seiberg–Witten map:

$$\widehat{\Lambda} = \Lambda + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu\Lambda\} + \ldots$$

$$\widehat{\psi} = \psi - \frac{1}{2}\theta^{\alpha\beta}\left(V_\alpha\,\partial_\beta - \frac{i}{4}[V_\alpha, V_\beta]\right)\psi + \ldots$$

$$\widehat{V}_\mu = V_\mu + \frac{1}{4}\theta^{\alpha\beta}\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\} + \ldots$$

GAUGE SECTOR

FRAMEWORK PROPOSAL

1: Commutative GFT, that are renormalizable are extended to the NC space with deformed gauge transformations. These deformations are not unique. For instance deformed action S_g depends on the choice of representation. This derives from the fact that $\widehat{F}^{\mu\nu}$ is enveloping algebra but not Lee algebra valued. The trace Tr in S_g is over all representations:

$$S_g = -\frac{1}{2}\,\mathrm{Tr}\int d^4x \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}.$$

2: Seiberg-Witten map up to 1st order in θ:

$$\widehat{V}_\mu(x) = V_\mu(x) - \frac{h}{4}\theta^{\rho\sigma}\left\{V_\rho(x), \partial_\sigma V_\mu(x) + F_{\sigma\mu}(x)\right\},$$

$$\widehat{F}_{\mu\nu} = F_{\mu\nu} + \frac{h}{4}\theta^{\rho\sigma}\Big(2\{F_{\rho\mu}, F_{\sigma\nu}\} - \{V_\rho, (\partial_\sigma + D_\sigma)F_{\mu\nu}\}\Big),$$

$$\widehat{F}_{\mu\nu} = \partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu - i[\widehat{V}_\mu \stackrel{\star}{,} \widehat{V}_\nu]$$

Points 1: and 2: leads to:

$$\begin{aligned} S_g^1 &= S_{cl} = S^0_{\mathrm{SM}} + S^\theta = -\frac{1}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}F^{\mu\nu} \\ &+ h\,\theta^{\rho\sigma}\mathrm{Tr}\int d^4x\, \left[\left(\frac{1}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right] \end{aligned}$$

Note: Solution for the SW map is NOT UNIQUE.
All expressions $\widehat{V}'_\mu$, $\widehat{F}'_{\mu\nu}$ of the form

$$\widehat{V}'_\mu = \widehat{V}_\mu + X_\mu, \quad \widehat{F}'_{\mu\nu} = \widehat{F}_{\mu\nu} + D_\mu X_\nu - D_\nu X_\mu,$$

are solutions to the closing condition to linear order, if X_μ is a gauge covariant expression linear in θ, otherwise arbitrary. This transformation is a redefinition of the fields V_μ and $F_{\mu\nu}$.

3: Clearly we do not know the meaning of 'minimal coupling concept' for some NCGFT in the NC space. However, renormalization is the principle that help us to find such acceptable couplings. We learned that the renormalizability condition of some specific NCGFT requires introduction of the higher order NC gauge interaction by expanding general NC action in terms of NC field strengths. This lead us to the extension of 'minimal' action S_g to higher order

$$S_\star = \mathrm{Tr}\int \mathrm{d}^4x \Big[-\frac{1}{2}\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu} + h\theta^{\mu\nu} b \widehat{F}_{\mu\nu}\star\widehat{F}_{\rho\sigma}\star\widehat{F}^{\rho\sigma}\Big],$$

with b being free parameter determining renormalizable deformation.

4: SW map for NC field strength up to the first order in $h\theta^{\mu\nu}$ gives

$$\begin{aligned} S_\star &= \mathrm{Tr}\int \mathrm{d}^4x \Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + h\theta^{\mu\nu}\left(\frac{1}{4}F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\right)F^{\rho\sigma} \\ &\qquad + h\theta^{\mu\nu}\, bF_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\Big], \\ &= \mathrm{Tr}\int \mathrm{d}^4x \Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} \\ &\qquad + h\theta^{\mu\nu}\left((\frac{1}{4}+b)F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\right)F^{\rho\sigma}\Big]. \end{aligned}$$

Redefinition $1+4b=a$:

$$S = \mathrm{Tr}\int \mathrm{d}^4x \Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + h\theta^{\mu\nu}\left(\frac{a}{4}F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\right)F^{\rho\sigma}\Big]$$

Proposed framework 1,...,4 gives starting action

$$S^a_\star = -\frac{1}{2}\mathrm{Tr}\int d^4x \left(1 - \frac{a-1}{2}\, h\, \theta^{\mu\nu} \star \widehat{F}_{\mu\nu}\right) \star \widehat{F}_{\rho\sigma} \star \widehat{F}^{\rho\sigma}\,,$$

which, via SW map for $\widehat{F}^{\mu\nu}$, produces directly:

$$\begin{aligned} S_{gauge} &= S_{cl} = S^0_{\mathrm{SM}} + S^\theta = -\frac{1}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}F^{\mu\nu} \\ &+ h\,\theta^{\mu\nu}\,\mathrm{Tr}\int d^4x \left(\frac{a}{4} F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\right)F^{\rho\sigma}. \end{aligned}$$

REQUIREMENT OF RENORMALIZABILITY
fixes the freedom parameter $a \Rightarrow$
PRINCIPLE OF RENORMALIZATION
DETERMINES
NC RENORMALIZABLE DEFORMATION

Trace of three generators in the above action lead to dependence of the gauge group representation!
The choice of the trace corresponds to the choice of the representation of the gauge group

Choosing however vector field in the adjoint representation, i.e. using a sum of three traces over the SM gauge group we have:
$\Rightarrow$ Model 1: mNCSM
Choosing a trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them we found:
$\Rightarrow$ Model 2: nmNCSM
Choosing however vector field in the adjoint representation SU(N) we have:
$\Rightarrow$ Model 3: NC SU(N) GFT

Gauge sector Model 1: mNCSM

Short rewiev of mNCSM gauge sector

The mNCSM gauge action is given by

$$S_{\text{gauge}}^{\text{mNCSM}} = -\frac{1}{2}\int d^4x \left(\frac{1}{g'^2}\text{Tr}_{\mathbf{1}} + \frac{1}{g^2}\text{Tr}_{\mathbf{2}} + \frac{1}{g_s^2}\text{Tr}_{\mathbf{3}}\right) \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}\,.$$

In the definition of $\text{Tr}_{\mathbf{1}}$:

$$Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The fundamental representations for $SU(2)$ and $SU(3)$ generators in $\text{Tr}_{\mathbf{2}}$ and $\text{Tr}_{\mathbf{3}}$, respectively. In terms of physical fields, the action then reads

$$\begin{aligned} S_{\text{gauge}}^{\text{mNCSM}} &= -\frac{1}{2}\int d^4x \left(\frac{1}{2}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + \text{Tr}\,\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} + \text{Tr}\,G_{\mu\nu}G^{\mu\nu}\right) \\ &\quad + \frac{1}{4}\,g_s\,d^{abc}\,\theta^{\rho\sigma}\int d^4x \left(\frac{a}{4}G^a_{\rho\sigma}G^b_{\mu\nu} - G^a_{\rho\mu}G^b_{\sigma\nu}\right) G^{\mu\nu,c}\,, \end{aligned}$$

where $\mathcal{A}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}(= B^a_{\mu\nu}T^a_L)$ and $G_{\mu\nu}(= G^a_{\mu\nu}T^a_S)$ denote the $U(1)$, $SU(2)_L$ and $SU(3)_c$ field strengths, respectively:

$$\begin{aligned} \mathcal{A}_{\mu\nu} &= \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu\,, \\ \mathcal{B}^a_{\mu\nu} &= \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\,\epsilon^{abc}B^b_\mu B^c_\nu\,, \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s\,f^{abc}G^b_\mu G^c_\nu\,. \end{aligned}$$

For adjoint representation $\Rightarrow$

* NO NEW NEUTRAL EW TGB INTERACTIONS

Gauge sector Model 2: nmNCSM

The action $S_{\text{gauge}}^{\text{nmNCSM}}$ up to linear order in θ:

$$
\begin{aligned}
S_{\text{gauge}}^{\text{nmNCSM}} &= S_{cl} = S^0_{\text{SM}} + S^\theta = -\frac{1}{2}\int d^4x\, \mathbf{Tr}\frac{1}{\mathbf{G}^2} F_{\mu\nu}F^{\mu\nu} \\
&+\theta^{\rho\sigma}\int d^4x\, \mathbf{Tr}\frac{1}{\mathbf{G}^2}\left[\left(\frac{a}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right]
\end{aligned}
$$

where $\mathbf{Tr}\frac{1}{\mathbf{G}^2}$ is trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them; 5 multiplets for each generation of fermions and 1 Higgs multiplet (Table). Here $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$ is SM field strenght, i.e. V_μ is the SM gauge potential:

$$
V^\mu = g' \mathcal{A}^\mu(x)Y + g\sum_{a=1}^{3} B_a^\mu(x)T_L^a + g_s\sum_{b=1}^{8} G_b^\mu(x)T_S^b
$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	T_3
$e_R^{(i)}$	$\mathbf{1}$	$\mathbf{1}$	-1	-1	0
$L_L^{(i)} = \begin{pmatrix} \nu_L^{(i)} \\ e_L^{(i)} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$u_R^{(i)}$	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$2/3$	0
$d_R^{(i)}$	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$-1/3$	0
$Q_L^{(i)} = \begin{pmatrix} u_L^{(i)} \\ d_L^{(i)} \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
W^+, W^-, Z	$\mathbf{1}$	$\mathbf{3}$	0	$(\pm1, 0)$	$(\pm1, 0)$
A	$\mathbf{1}$	$\mathbf{1}$	0	0	0
G^b	$\mathbf{8}$	$\mathbf{1}$	0	0	0

The SM fields. Here $i \in \{1,2,3\}$ denotes the generation index. The electric charge is given by the Gell-Mann-Nishijima relation $Q = (T_3 + Y)$. The physical electroweak fields A, W^+, W^- and Z are expressed through the unphysical $U(1)_Y$ and $SU(2)$ fields $\mathcal{A}$ and B_a ($a \in \{1,2,3\}$). The gluons G^b ($b \in \{1,2,\ldots,8\}$) are in the octet representation of $SU(3)_C$.

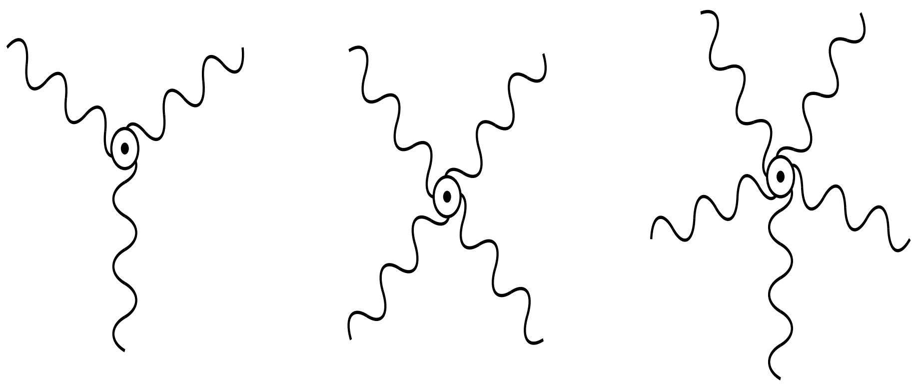
For SM gauge group we denote field strenght as:

$$F^{\mu\nu} = g' f^{\mu\nu}\mathcal{R}(Y) + g \sum_{a=1}^{3} B_a^{\mu\nu}\mathcal{R}(T_L^a) + g_s \sum_{b=1}^{8} G_b^{\mu\nu}\mathcal{R}(T_S^b)$$

Lagrangian linear correction in θ has trace of products of 3 field strenghts. Written genericaly that is:

$$\begin{aligned}
F^3 \sim\ & g'^3\, f^3\, Tr\mathcal{R}(Y)^3\, TrI\, TrI \neq 0 \\
+\ & g^3\, B^3\, Tr\mathcal{R}(T^i)^3\, TrI \qquad \sim d^{ijk} \text{ for SU(2)} \\
+\ & {g_s}^3\, G^3\, TrI\, Tr\mathcal{R}(T^a)^3\, TrI \qquad \sim d^{abc} \text{ for SU(3)} \\
+\ & {g_s}^3\, f^2 B\, TrT^i\, TrI = 0 \\
+\ & g' g^2\, f B^2\, Tr(T^i)^2\, TrI \neq 0 \\
+\ & g'^2 g_s\, f G^2\, TrI\, Tr(T^a)^2 = 0 \\
+\ & g' g_s^2\, f G^2\, TrI\, Tr(T^a)^2\, TrI \neq 0 \\
+\ & g {g_s}^2\, B G^2\, TrT^i\, Tr(T^a)^2 = 0 \\
+\ & g^2 g_s\, B^2 G\, Tr(T^i)^2\, TrT^a = 0
\end{aligned}$$

Nonzero are only 3 terms containinig 3, 4 and 5 fields linear in θ



The NC couplings $\rightarrow$ additional vertices.
The lines are gauge fields $\mathcal{A}_\mu$, B_μ^i and G_μ^a

Matching the SM action at zeroth order in θ, three consistency conditions are imposed

$$\begin{aligned}
\frac{1}{g'^2} &= \frac{2}{g_1^2}+\frac{1}{g_2^2}+\frac{8}{3g_3^2}+\frac{2}{3g_4^2}+\frac{1}{3g_5^2}+\frac{1}{g_6^2},\\
\frac{1}{g^2} &= \frac{1}{g_2^2}+\frac{3}{g_5^2}+\frac{1}{g_6^2},\\
\frac{1}{g_s^2} &= \frac{1}{g_3^2}+\frac{1}{g_4^2}+\frac{2}{g_5^2}.
\end{aligned}$$

giving final expression for TGB action

$$\begin{aligned}
S_{gauge} &= S_{cl} = S^0_{\mathrm{SM}} + S^\theta = -\frac{1}{4}\int d^4x f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\int d^4x \,\mathrm{Tr}\,(B_{\mu\nu}B^{\mu\nu})\\
&- \frac{1}{2}\int d^4x \,\mathrm{Tr}\,(G_{\mu\nu}G^{\mu\nu})\\
&+ g'^2\kappa_1\theta^{\rho\tau}\int d^4x \left(\frac{a}{4}f_{\rho\tau}f_{\mu\nu} - f_{\mu\rho}f_{\nu\tau}\right) f^{\mu\nu}\\
&+ g' g^2\kappa_2\,\theta^{\rho\tau}\int d^4x \sum_{a=1}^{3}\left[(\frac{a}{4}f_{\rho\tau}B^a_{\mu\nu} - f_{\mu\rho}B^a_{\nu\tau})B^{\mu\nu,a} + c.p.\right]\\
&+ g' g_s^2\kappa_3\,\theta^{\rho\tau}\int d^4x \sum_{b=1}^{8}\left[(\frac{a}{4}f_{\rho\tau}G^b_{\mu\nu} - f_{\mu\rho}G^b_{\nu\tau})G^{\mu\nu,b} + c.p.\right]
\end{aligned}$$

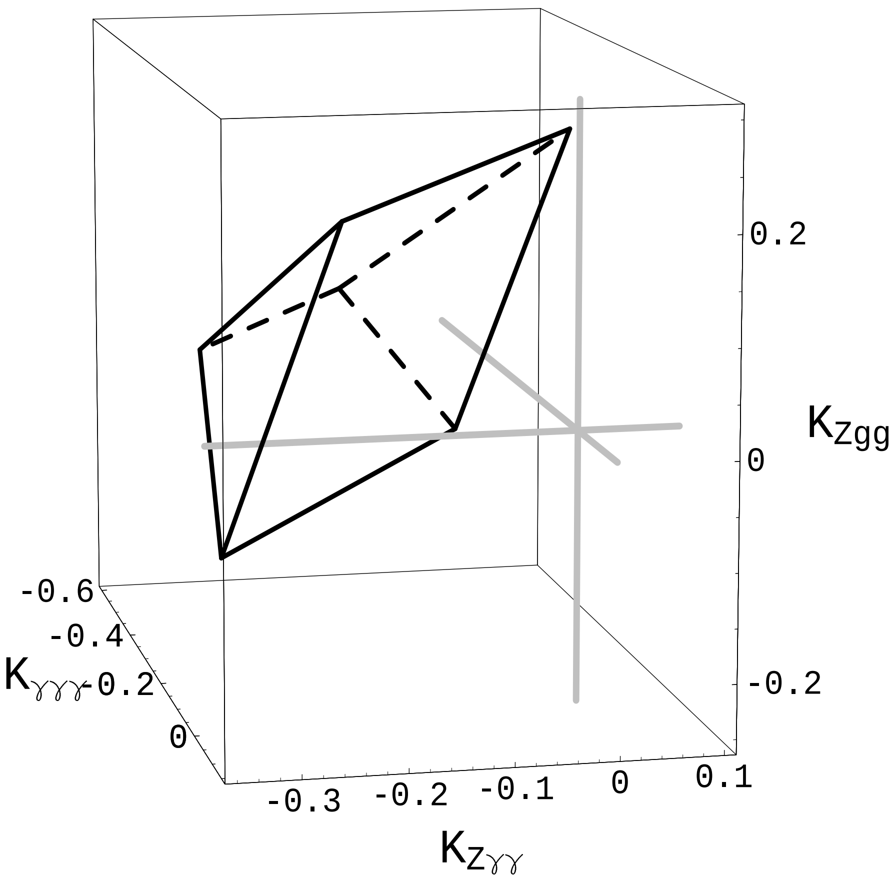
Above three consistenty conditions together with the requirement that $1/g_i^2 > 0$ define a 3D pentahedron in the six-dimensional moduli space spanned by $1/g_1^2, ..., 1/g_6^2$

$$\frac{2K_{\gamma\gamma\gamma}}{gg'} = -\frac{1}{g_1^2} - \frac{1}{g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \frac{7}{9g_5^2} + \frac{1}{g_6^2},$$

$$\frac{2K_{Z\gamma\gamma}}{g'^2} = -\frac{1}{g_1^2} - \left(1 - (\frac{g}{g'})^2\right)\frac{1}{2g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2}$$

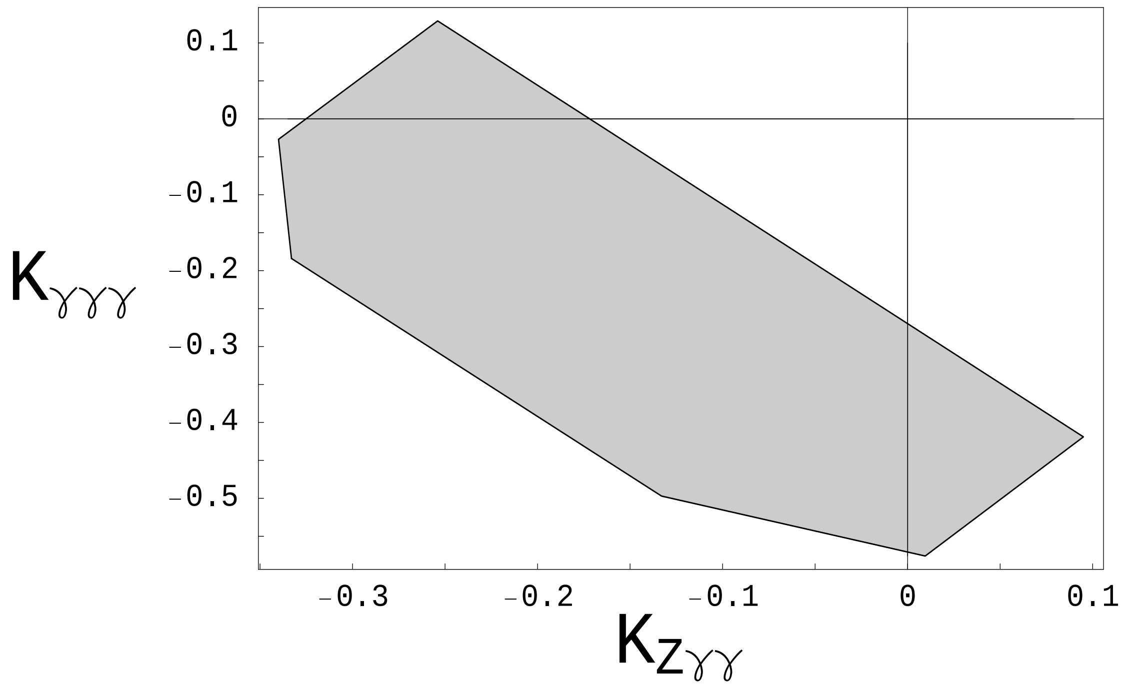
$$+ \left(5 - 9(\frac{g}{g'})^2\right)\frac{1}{18g_5^2} + \left(1 - (\frac{g}{g'})^2\right)\frac{1}{2g_6^2},$$

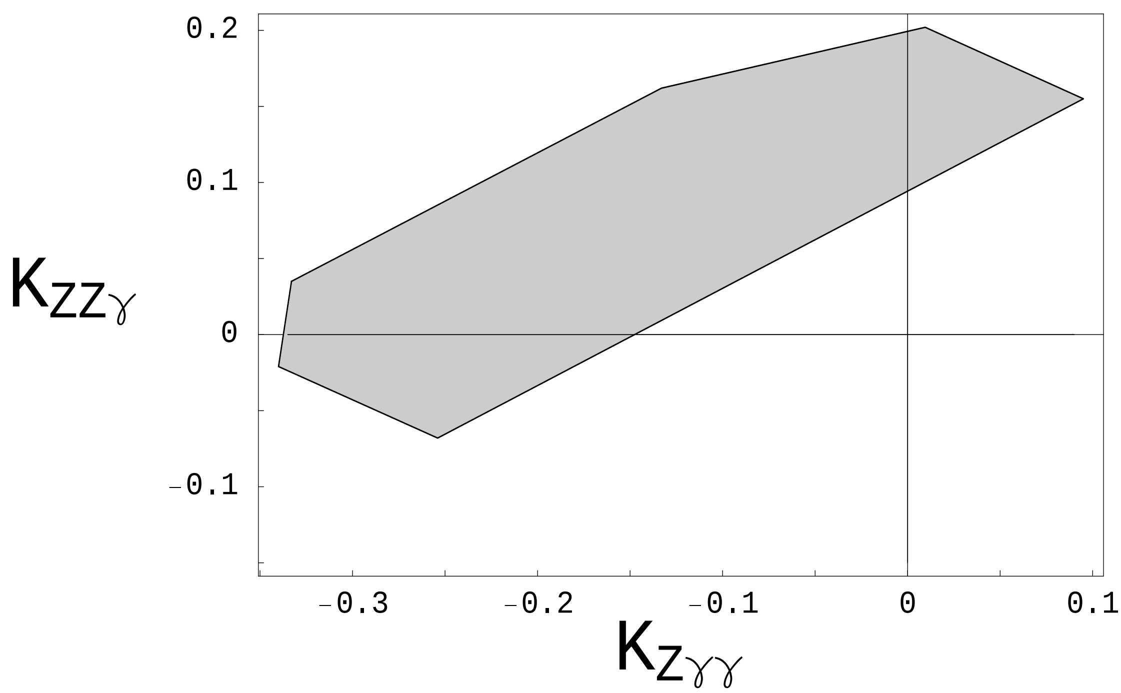
$$\frac{2K_{Zgg}}{g_s^2} = \left(1 + (\frac{g'}{g})^2\right)\left(\frac{1}{3g_3^2} - \frac{1}{6g_4^2} + \frac{1}{6g_5^2}\right).$$



$K_{\gamma\gamma\gamma}$	$K_{Z\gamma\gamma}$	K_{Zgg}	$K_{ZZ\gamma}$	K_{ZZZ}	$K_{\gamma gg}$
−0.184	−0.333	0.054	0.035	−0.213	−0.098
−0.027	−0.340	−0.108	−0.021	−0.337	0.197
0.129	−0.254	0.217	−0.068	−0.362	−0.396
−0.576	0.010	−0.108	0.202	0.437	0.197
−0.497	−0.133	0.054	0.162	0.228	−0.098
−0.419	0.095	0.217	0.155	0.410	−0.396

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]





The interactions $\mathcal{L}^\theta$ in terms of physical fields (A, Z, W, G)

$$\mathcal{L}^\theta_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W \, \mathsf{K}_{\gamma\gamma\gamma}\theta^{\rho\tau}A^{\mu\nu}\left(aA_{\mu\nu}A_{\rho\tau} - 4A_{\mu\rho}A_{\nu\tau}\right)$$

$$\mathsf{K}_{\gamma\gamma\gamma} = \frac{1}{2}\, gg'(\kappa_1 + 3\kappa_2)$$

$$\mathcal{L}^\theta_{Z\gamma\gamma} = \frac{e}{4}\sin 2\theta_W \, \mathsf{K}_{Z\gamma\gamma}\,\theta^{\rho\tau}\,[2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\tau} - aA_{\mu\nu}A_{\rho\tau}\right) + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - aZ_{\rho\tau}A_{\mu\nu}A^{\mu\nu}]$$

$$\mathsf{K}_{Z\gamma\gamma} = \frac{1}{2}\left[{g'}^2\kappa_1 + \left({g'}^2 - 2g^2\right)\kappa_2\right]$$

$$\mathcal{L}^\theta_{WW\gamma} = \frac{e}{4}\sin 2\theta_W \, \mathsf{K}_{WW\gamma}\,\theta^{\rho\tau}\left\{A^{\mu\nu}\left[2\left(W^+{}_{\mu\rho}W^-{}_{\nu\tau} + W^-{}_{\mu\rho}W^+{}_{\nu\tau}\right) - a\left(W^+{}_{\mu\nu}W^-{}_{\rho\tau} + W^-{}_{\mu\nu}W^+{}_{\rho\tau}\right)\right] + 4A_{\mu\rho}\left(W^{+\mu\nu}W^-{}_{\nu\tau} + W^{-\mu\nu}W^+{}_{\nu\tau}\right) - aA_{\rho\tau}W^+{}_{\mu\nu}W^{-\mu\nu}\right]\right\}$$

$$\mathsf{K}_{WW\gamma} = -\frac{g}{g'}\left[{g'}^2 + g^2\right]\kappa_2$$

$$\mathcal{L}^\theta_{WWZ} = \mathcal{L}_{WW\gamma}(A \leftrightarrow Z)$$

$$\mathsf{K}_{WWZ} = -\frac{g'}{g}\,\mathsf{K}_{WW\gamma}$$

$$\mathcal{L}^\theta_{ZZ\gamma} = \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z)$$

$$\mathsf{K}_{ZZ\gamma} = \frac{-1}{2gg'}\left[{g'}^4\kappa_1 + g^2\left(g^2 - 2{g'}^2\right)\kappa_2\right]$$

$$\mathcal{L}^\theta_{ZZZ} = \mathcal{L}_{\gamma\gamma\gamma}(A \to Z)$$

$$\mathsf{K}_{ZZZ} = \frac{-1}{2g^2}\left[{g'}^4\kappa_1 + 3g^4\kappa_2\right]$$

$$\mathcal{L}^\theta_{Zgg} = \mathcal{L}_{Z\gamma\gamma}(A \to G^b)$$

$$\mathsf{K}_{Zgg} = \frac{g_s^2}{2}\left[1 + (\frac{g'}{g})^2\right]\kappa_3$$

$$\mathcal{L}^\theta_{\gamma gg} = \mathcal{L}_{Zgg}(Z \to A)$$

$$\mathsf{K}_{\gamma gg} = \frac{-g_s^2}{2}\left[\frac{g}{g'} + \frac{g'}{g}\right]\kappa_3; \quad A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \ldots$$

Renormalization Model 2: nmNCSM

⋆ Advantage of the background field method is the guarantee of covariance, because by doing the path integral the local symmetry of the quantum field Φ_V is fixed, while the gauge symmetry of the background field ϕ_V is manifestly preserved.

⋆ Quantization is performed by the functional integration over the quantum vector field Φ_V in the saddle-point approximation around classical (background) configuration. Our case $\phi_V = constant$.

⋆ The main contribution to the functional integral is given by the Gaussian integral.

⋆ Split the vector potential into the classical background plus the quantum-fluctuation parts, that is: We replace, $\phi_V \to \phi_V + \Phi_V$, and than compute the terms quadratic in the quantum fields.

⋆ Interactions are of the polynomial type.

⋆ Proper quantization requires the presence of the gauge fixing term $S_{\mathsf{gf}}[\phi]$. Adding to the SM part in the usual way, FFP ghost appears in the effective action. Result of functional integration

$$\begin{aligned}\Gamma[\phi] &= S_{\mathsf{cl}}[\phi] + S_{\mathsf{gf}}[\phi] + \Gamma^{(1)}[\phi],\\ S_{\mathsf{gf}}[\phi] &= -\frac{1}{2}\int \mathsf{d}^4x (D_\mu \phi_V^\mu)^2\,,\end{aligned}$$

produce the standard result of the commutative part of our action:

$$\Gamma^{(1)}[\phi] = \frac{\mathsf{i}}{2}\log\det S^{(2)}[\phi] = \frac{\mathsf{i}}{2}\mathsf{Tr}\,\log S^{(2)}[\phi].$$

The $S^{(2)}[\phi]$ is the 2^{nd}-functional derivative of the classical action,

$$S^{(2)}[\phi] = \frac{\delta^2 S_{\text{cl}}}{\delta\phi_{V_1}\delta\phi_{V_2}}.$$

After making the splitting

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \mathbf{A}_\mu, \quad B^i_\mu \to B^i_\mu + \mathbf{B}^i_\mu, \quad G^a_\mu \to G^a_\mu + \mathbf{G}^a_\mu,$$

we obtain for the quadratic part of the action :

$$\frac{1}{2}\left(\mathbf{A}_\alpha\,\mathbf{B}^i_\alpha\,\mathbf{G}^a_\alpha\right)\begin{pmatrix} g^{\alpha\beta}\Box + M^{\alpha\beta} & * & * \\ * & g^{\alpha\beta}\delta^{ij}\Box + V^{\alpha\beta;ij} & 0 \\ * & 0 & g^{\alpha\beta}\delta^{ab}\Box + W^{\alpha\beta ab}\end{pmatrix}\begin{pmatrix}\mathbf{A}_\beta \\ \mathbf{B}^j_\beta \\ \mathbf{G}^b_\beta\end{pmatrix}.$$

$\Box$ - propagator of any field

$*$ - terms which will not contribute θ^1: they give only higher-order corrections.

$$M^{\alpha\beta} = \overleftarrow{\partial_\mu} M^{\mu\alpha,\nu\beta}(x)\overrightarrow{\partial_\nu}$$

$$\begin{aligned} M^{\mu\rho,\nu\sigma} &= \frac{1}{2}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho})\theta^{\alpha\beta}f_{\alpha\beta} \\ &+ g^{\mu\nu}(\theta^{\alpha\rho}f^{\sigma}{}_{\alpha} + \theta^{\alpha\sigma}f^{\rho}{}_{\alpha}) + g^{\rho\sigma}(\theta^{\alpha\mu}f^{\nu}{}_{\alpha} + \theta^{\alpha\nu}f^{\mu}{}_{\alpha}) \\ &- g^{\mu\sigma}(\theta^{\alpha\rho}f^{\nu}{}_{\alpha} + \theta^{\alpha\nu}f^{\rho}{}_{\alpha}) - g^{\nu\rho}(\theta^{\alpha\sigma}f^{\mu}{}_{\alpha} + \theta^{\alpha\mu}f^{\sigma}{}_{\alpha}) \\ &+ \theta^{\mu\rho}f^{\nu\sigma} + \theta^{\nu\sigma}f^{\mu\rho} - \theta^{\rho\sigma}f^{\mu\nu} - \theta^{\mu\nu}f^{\rho\sigma} - \theta^{\nu\rho}f^{\mu\sigma} - \theta^{\mu\sigma}f^{\nu\rho} \end{aligned}$$

The structure of $V^{\alpha\beta;ij}$ is as follows:

$$V^{\alpha\beta;ij} = (N_1 + N_2 + T_1 + T_2 + T_3)^{\alpha\beta;ij}.$$

The operators N_1 and N_2 come from the commutative 3-vertex and 4-vertex interactions:

$$\begin{aligned} (N_1)^{ij}_{\alpha\beta} &= -2ig_{\alpha\beta}(B_\mu)^{ij}\partial^\mu - i(\partial^\mu B_\mu)^{ij}g_{\alpha\beta}, \\ (N_2)^{ij}_{\alpha\beta} &= -(B_\mu B^\mu)^{ij}g_{\alpha\beta} - 2i(B_{\alpha\beta})^{ij}, \end{aligned}$$

the notation $(X_\mu)^{ij} = -if^{ijk}X^k_\mu$. The operators T_1, T_2 and T_3 describe the θ^1, noncommutative vertices.

$$
\begin{aligned}
(T_1)^{ij}_{\alpha\beta} &= g'g^2\kappa_2\delta^{ij}\Big[a(\overleftarrow{\partial_\mu}\theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\rho\sigma}\overrightarrow{\partial_\alpha}) \\
&- 2(\overleftarrow{\partial_\beta}\theta_{\rho\alpha}f^{\mu\rho}\overrightarrow{\partial_\mu} - \overleftarrow{\partial^\nu}\theta^\rho{}_\alpha f_{\beta\rho}\overrightarrow{\partial_\nu} - \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\beta\rho}\overrightarrow{\partial_\alpha} \\
&+ \overleftarrow{\partial_\mu}\theta_{\rho\beta}f^{\mu\rho}\overrightarrow{\partial_\alpha} - \overleftarrow{\partial^\nu}\theta^\rho{}_\beta f_{\alpha\rho}\overrightarrow{\partial_\nu} - \overleftarrow{\partial^\mu}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial_\sigma} \\
&+ \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\alpha\rho}\overrightarrow{\partial_\sigma}) + 2a(\overleftarrow{\partial_\rho}\theta^\rho{}_\alpha f_{\mu\beta}\overrightarrow{\partial^\mu} + \overleftarrow{\partial^\mu}\theta^\rho{}_\beta f_{\mu\alpha}\overrightarrow{\partial_\rho}) \\
&- 2(\overleftarrow{\partial_\mu}\theta_{\alpha\beta}f^{\mu\nu}\overrightarrow{\partial_\nu} - \overleftarrow{\partial^\mu}\theta_{\alpha\sigma}f_{\mu\beta}\overrightarrow{\partial^\sigma} - \overleftarrow{\partial^\sigma}\theta_{\beta\sigma}f_{\mu\alpha}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\rho}\theta^{\rho\sigma}f_{\alpha\beta}\overrightarrow{\partial_\sigma})\Big], \\
(T_2)^{ij}_{\alpha\beta} &= g'g^2 i\kappa_2\Big[a(-\overleftarrow{\partial_\mu}\theta^{\rho\sigma}g_{\alpha\beta}f_{\rho\sigma}(B^\mu)^{ij} - \theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial_\mu} \\
&+ \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\rho\sigma}(B_\alpha)^{ij} + \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta)^{ji}\overrightarrow{\partial_\alpha} + \theta_{\rho\sigma}f^{\rho\sigma}(B_{\alpha\beta})^{ij}) \\
&- 2(-\overleftarrow{\partial_\beta}\theta_{\rho\alpha}f^{\mu\rho}(B_\mu)^{ij} - \theta_{\rho\beta}f^{\mu\rho}(B_\mu)^{ji}\overrightarrow{\partial_\alpha} + \overleftarrow{\partial_\nu}\theta_{\rho\alpha}f_\beta{}^\rho(B^\nu)^{ij} \\
&+ \theta_{\rho\beta}f_\alpha{}^\rho(B^\nu)^{ji}\overrightarrow{\partial_\nu} + \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial_\sigma} \\
&- \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\beta\rho}(B_\alpha)^{ij} - \theta^{\rho\sigma}f_{\alpha\rho}(B_\beta)^{ji}\overrightarrow{\partial_\sigma} - \overleftarrow{\partial_\mu}\theta_{\rho\beta}f^{\mu\rho}(B_\alpha)^{ij} - \theta_{\rho\alpha}f^{\mu\rho}(B_\beta)^{ji}\overrightarrow{\partial_\mu} \\
&+ \overleftarrow{\partial_\mu}\theta^{\rho\sigma}g_{\alpha\beta}f^\mu{}_\rho(B_\sigma)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B_\sigma)^{ji}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\mu}\theta^\rho{}_\beta f_{\alpha\rho}(B^\mu)^{ij} \\
&+ \theta_{\rho\alpha}f_\beta{}^\rho(B^\mu)^{ji}\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\alpha\rho}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma)^{ji}\overrightarrow{\partial_\alpha} + \theta^{\rho\sigma}f_{\alpha\rho}(B_{\beta\sigma})^{ij} \\
&+ \theta_{\rho\beta}f^{\mu\rho}(B_{\mu\alpha})^{ij} + \theta^{\rho\sigma}f_{\beta\rho}(B_{\alpha\sigma})^{ji} \\
&+ \theta_{\rho\alpha}f^{\mu\rho}(B^\mu{}_\beta)^{ji}) - 2a(\overleftarrow{\partial^\rho}\theta_{\rho\alpha}f_{\mu\beta}(B^\mu)^{ij} + \theta_{\rho\beta}f_{\mu\alpha}(B^\mu)^{ji}\overrightarrow{\partial^\rho} \\
&+ \overleftarrow{\partial^\mu}\theta_{\rho\beta}f_{\mu\alpha}(B^\rho)^{ij} + \theta_{\rho\alpha}f_{\mu\beta}(B^\rho)^{ji}\overrightarrow{\partial^\mu} - \frac{1}{2}\theta_{\rho\sigma}f_{\alpha\beta}(B^{\rho\sigma})^{ij} \\
&- \frac{1}{2}\theta_{\alpha\beta}f_{\rho\sigma}(B^{\rho\sigma})^{ij}) - 2(-\overleftarrow{\partial^\mu}\theta_{\alpha\beta}f_{\mu\nu}(B^\nu)^{ij} - \theta_{\beta\alpha}f_{\mu\nu}(B^\nu)^{ji}\overrightarrow{\partial^\mu} \\
&+ \overleftarrow{\partial^\mu}\theta_{\alpha\sigma}f_{\mu\beta}(B^\sigma)^{ij} + \theta_{\beta\sigma}f_{\mu\alpha}(B^\sigma)^{ji}\overrightarrow{\partial^\mu} + \overleftarrow{\partial^\rho}\theta_{\rho\beta}f_{\alpha\nu}(B^\nu)^{ij} + \theta_{\rho\alpha}f_{\beta\nu}(B^\nu)^{ji}\overrightarrow{\partial^\rho} \\
&- \overleftarrow{\partial_\rho}\theta_{\beta\sigma}f_{\alpha\beta}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\alpha}(B_\sigma)^{ji}\overrightarrow{\partial_\rho} + \theta_{\beta\sigma}f_{\alpha\nu}(B^{\nu\sigma})^{ij} + \theta_{\alpha\sigma}f_{\beta\nu}(B^\nu{}_\sigma)^{ji})\Big], \\
(T_3)^{ij}_{\alpha\beta} &= g'g^2\kappa_2\Big[a(\theta^{\rho\sigma}f_{\rho\sigma}(B_\mu B^\mu)^{ij}g_{\alpha\beta} - \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta B_\alpha)^{ij}) \\
&- 2(\theta_{\rho\alpha}f^{\mu\rho}(B_\beta B_\mu)^{ij} - \theta^\rho{}_\alpha f_{\beta\rho}(B_\nu B^\nu)^{ij} - \theta^{\rho\sigma}f_{\mu\rho}(B_\sigma B^\mu)^{ij}g_{\alpha\beta} \\
&+ \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma B_\alpha)^{ij} + (\alpha\leftrightarrow\beta \quad i\leftrightarrow j)) \\
&+ 2a(\theta_{\rho\alpha}f_{\mu\beta}(B^\rho B^\mu)^{ij} + 2\theta_{\rho\beta}f_{\mu\alpha}(B^\rho B^\mu)^{ji}) \\
&- 2((\theta_{\alpha\beta}f^{\mu\nu}(B_\mu B_\nu)^{ij} \\
&- \theta_{\alpha\sigma}f_{\mu\beta}(B^\mu B^\sigma)^{ij} - \theta_{\beta\sigma}f_{\mu\alpha}(B^\mu B^\sigma)^{ji} + \theta^{\rho\sigma}f_{\alpha\beta}(B_\rho B_\sigma)^{ij})\Big].
\end{aligned}
$$

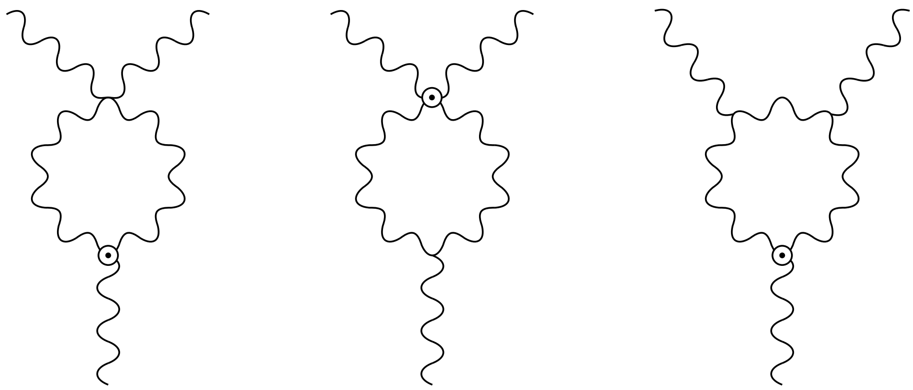
The matrix $W^{\alpha\beta,ab}$ analogous to $V^{\alpha\beta,ij}$ up to the change $B^i_\mu \leftrightarrow G^a_\mu$.

The one-loop effective action is

$$\begin{aligned}\Gamma^{(1)}_{\theta,2} &= \frac{i}{2}\mathrm{Tr}\,\log\left(\mathcal{I}+\Box^{-1}(N_1+N_2+T_1+T_2+T_3)\right)\\ &= \frac{i}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{Tr}\left(\Box^{-1}N_1+\Box^{-1}N_2+\Box^{-1}T_1+\Box^{-1}T_2+\Box^{-1}T_3\right)^n.\end{aligned}$$

the divergences in θ-linear order are all of the form $\theta f B^2$. Need to extract and compute only terms that contain three external fields.

$$\Gamma^{(1)}_{\theta,2} = \frac{i}{2}\mathrm{Tr}\,[(\Box^{-1}N_1)^2\Box^{-1}T_1 - \Box^{-1}N_1\Box^{-1}T_2 - \Box^{-1}N_2\Box^{-1}T_1].$$



1-loop divergent corrections to the θ-3-vertex also contains the contributions to the θ-4-vertex and θ-5-vertex.

Computed divergences due to the $U(1)_Y - SU(2)_L$ part of the noncommutative action, S_2^θ using background field method; divergent part calculated in momentum representation by dimensional regularization.

$$\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_1\Box^{-1}T_2) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2 \\
&\times \Big[(6-2a)(\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})(B^{\alpha i}\partial_\mu\partial_\sigma B^{\mu i}-B^{\alpha i}\Box B^i_\sigma) \\
&+ (3a-4)\theta^{\rho\sigma}f_{\rho\sigma}(B^{\nu i}\partial_\mu\partial_\nu B^{\mu i}-B^i_\mu\Box B^{\mu i})\Big],
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_2\Box^{-1}T_1) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2 \\
&\times \Big[(2a-6)(\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})(B^{\nu i}\partial_\sigma\partial^\alpha B^i_\nu+\partial_\sigma B^{\mu i}\partial^\alpha B^i_\mu) \\
&+ \theta^{\rho\sigma}f_{\rho\sigma}(18-11a)(\partial_\nu B^{\nu i}\partial_\mu B^{\mu i}+B^i_\mu\Box B^{\mu i})\Big],
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_1^2\Box^{-1}T_1) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2\Big[\theta^{\rho\sigma}f_{\rho\sigma}\Big((22-14a)B^i_\mu\Box B^{\mu i} \\
&+ (15-10a)\partial_\nu B^{\mu i}\partial^\nu B^i_\mu \\
&+ (3a-4)B^{\mu i}\partial_\mu\partial_\nu B^{\nu i}+(3-a)\partial_\mu B^{\nu i}\partial_\nu B^{\mu i}\Big) \\
&+ (\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})\Big((2a-6)(B^i_\sigma\Box B^{\alpha i}-B^i_\sigma\partial^\alpha\partial_\mu B^{\mu i} \\
&+ B^{\mu i}\partial_\sigma\partial^\alpha B^i_\mu-\partial_\sigma B^{\mu i}\partial_\mu B^{\alpha i})+(a-3)\partial_\mu B^{\alpha i}\partial^\mu B^i_\sigma \\
&+ (3a-9)\partial_\sigma B^{\mu i}\partial^\alpha B^i_\mu\Big)\Big].
\end{aligned}$$

The result for $U(1)_Y - SU(3)_C$ is analogous and follows immediately. Finally

$$\begin{aligned}
\Gamma^{(1)}_{\mathrm{div}} &= \frac{11}{3(4\pi)^2\epsilon}\int d^4x B^i_{\mu\nu}B^{\mu\nu i}+\frac{11}{2(4\pi)^2\epsilon}\int d^4x G^a_{\mu\nu}G^{\mu\nu a} \\
&+ \frac{4}{3(4\pi)^2\epsilon}g'g^2\kappa_2(3-a)\theta^{\mu\nu}\int d^4x\Big(\frac{1}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i}-f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i}\Big) \\
&+ \frac{6}{3(4\pi)^2\epsilon}g'g_S^2\kappa_3(3-a)\theta^{\mu\nu}\int d^4x\Big(\frac{1}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a}-f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a}\Big).
\end{aligned}$$

Renormalization via Counterterms & $a = 3$

$$
\begin{aligned}
\mathcal{L} + \mathcal{L}_{ct} &= -\frac{1}{4} f_{0\mu\nu} f_0^{\mu\nu} - \frac{1}{4} B_{0\mu\nu}^{i} B_0^{\mu\nu i} - \frac{1}{4} G_{0\mu\nu}^{a} G_0^{\mu\nu a} \\
&+ g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} f_{0\rho\sigma} f_0^{\rho\sigma} - f_{0\mu\rho} f_{0\nu\sigma} f_0^{\rho\sigma} \right) \\
&+ g'_0 g_0^2 \kappa_2 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} B_{0\rho\sigma}^{i} B_0^{\rho\sigma i} - f_{0\mu\rho} B_{0\nu\sigma}^{i} B_0^{\rho\sigma i} + c.p. \right) \\
&+ g'_0 (g_S)_0^2 \kappa_3 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} G_{0\rho\sigma}^{a} G_0^{\rho\sigma a} - f_{0\mu\rho} G_{0\nu\sigma}^{a} G_0^{\rho\sigma a} + c.p. \right),
\end{aligned}
$$

Bare quantities are:

$$
\begin{aligned}
\mathcal{A}_0^{\mu} &= \mathcal{A}^{\mu}, \qquad g'_0 = g', \\
B_0^{\mu i} &= B^{\mu i} \sqrt{1 + \frac{44 g^2}{3(4\pi)^2 \epsilon}}, \quad g_0 = \frac{g\,\mu^{\epsilon/2}}{\sqrt{1 + \frac{44 g^2}{3(4\pi)^2\epsilon}}}, \\
G_0^{\mu a} &= G^{\mu a} \sqrt{1 + \frac{22 g_S^2}{(4\pi)^2 \epsilon}}, \quad (g_S)_0 = \frac{g_S\,\mu^{\epsilon/2}}{\sqrt{1 + \frac{22 g_S^2}{(4\pi)^2\epsilon}}}.
\end{aligned}
$$

κ_1, κ_2 and κ_3 unchanged under renormalization

$$
\kappa_1 = (\kappa_1)_0, \quad \kappa_2 = (\kappa_2)_0, \quad \kappa_3 = (\kappa_3)_0,
$$

replacement:

$$
\begin{aligned}
\frac{1}{g_1^2} &= \left(\frac{1}{g_1^2}\right)_0 + \frac{33}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_2^2} = \left(\frac{1}{g_2^2}\right)_0 + \frac{-11}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_3^2} = \left(\frac{1}{g_3^2}\right)_0 + \frac{-11}{18(4\pi)^2\epsilon}, \\
\frac{1}{g_4^2} &= \left(\frac{1}{g_4^2}\right)_0 + \frac{-143}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_5^2} = \left(\frac{1}{g_5^2}\right)_0 + \frac{-121}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_6^2} = \left(\frac{1}{g_6^2}\right)_0 + \frac{110}{18(4\pi)^2\epsilon}.
\end{aligned}
$$

NC parameter θ need not be renormalized

because $\mathcal{L}^{\theta}$ is free from divergences.

Gauge sector Model 3: NC SU(N) GFT

$$S_{cl} = S_{\text{NCYM}} = \int d^4x \left(-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{4} h\theta^{\mu\nu} d^{abc} \left(\frac{a}{4} F^a_{\mu\nu} F^b_{\rho\sigma} - F^a_{\mu\rho} F^b_{\nu\sigma} \right) F^{c\rho\sigma} \right),$$

Here earlier introduced noncommutativity deformation parameter h becomes very important.

Renormalization:

Second functional derivative $S^2[\phi]$ of S_{cl}

$$S^2 = \Box + N_1 + N_2 + T_2 + T_3 + T_4 \ ,$$

N_1, N_2 - commutative vertices
T_2, T_3, T_4 non-commutative vertices

The 1-loop effective action computed by using BFM

$$\Gamma^{(1)}_{\theta,2} = \frac{i}{2} \text{Tr} \log \left(\mathcal{I} + \Box^{-1}(N_1 + N_2 + T_2 + T_3 + T_4) \right)$$
$$= \frac{i}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{Tr} \left(\Box^{-1} N_1 + \Box^{-1} N_2 + \Box^{-1} T_2 + \Box^{-1} T_3 + \Box^{-1} T_4 \right)^n .$$

Vertices are

$$(N_1)^{ab\alpha\beta} = -2i(A^\mu)^{ab} g^{\alpha\beta} \partial_\mu,$$
$$(N_2)^{ab\alpha\beta} = -2f^{abc} F^{c\alpha\beta} - (A^\mu A_\mu)^{ab} g^{\alpha\beta},$$

Noncommutative vertices are

$$
\begin{aligned}
(T_2)^{ab\alpha\beta} &= \frac{h}{8} d^{abc} \Big\{ \Big[\Big(a\theta^{\rho\sigma} F^c_{\rho\sigma} g^{\alpha\nu} g^{\beta\mu} - 2(a-1)\theta^{\alpha\mu} F^{c\beta\nu} + 4\theta^{\alpha}{}_{\rho} F^{c\beta\rho} g^{\mu\nu} \\
&\quad + 4\theta^{\mu}{}_{\rho} F^{c\nu\rho} g^{\alpha\beta} \Big) - (\beta \leftrightarrow \nu) \Big] + [\alpha \leftrightarrow \beta] \Big\} \partial_\mu \partial_\nu \,, \\
(T_3)^{ab\alpha\beta} &= \frac{\mathrm{i}h}{4} \Big\{ d^{acd} \Big[-2a\theta^{\alpha\mu} (A_\nu)^{bc} F^{d\beta\nu} - 2a\theta^{\beta\nu} (A_\nu)^{bc} F^{d\alpha\mu} \\
&\quad - a\theta^{\rho\sigma} (A^\mu)^{bc} F^d_{\rho\sigma} g^{\alpha\beta} + a\theta^{\rho\sigma} (A^\alpha)^{bc} F^d_{\rho\sigma} g^{\beta\mu} \\
&\quad - 2\theta^{\alpha}{}_{\rho} (A_\nu)^{bc} F^{d\nu\rho} g^{\beta\mu} + 2\theta^{\alpha}{}_{\nu} (A^\mu)^{bc} F^{d\beta\nu} \\
&\quad + 2\theta^{\mu\rho} (A^\nu)^{bc} F^d_{\nu\rho} - 2\theta^{\mu}{}_{\rho} (A^\alpha)^{bc} F^{d\beta\rho} - 2\theta^{\beta}{}_{\rho} (A^\alpha)^{bc} F^{d\mu\rho} \\
&\quad + 2\theta^{\beta}{}_{\rho} (A^\mu)^{bc} F^{d\alpha\rho} + 2\theta^{\nu}{}_{\rho} (A_\nu)^{bc} F^{d\mu\rho} g^{\alpha\beta} \\
&\quad + 2\theta^{\alpha\beta} (A_\nu)^{bc} F^{d\mu\nu} + 2\theta^{\alpha\nu} (A_\nu)^{bc} F^{d\beta\mu} + 2\theta^{\beta\mu} (A_\nu)^{bc} F^{d\alpha\nu} \\
&\quad - 2\theta^{\nu}{}_{\rho} (A_\nu)^{bc} F^{d\alpha\rho} g^{\beta\mu} + 2\theta^{\mu\nu} (A_\nu)^{bc} F^{d\alpha\beta} \Big] \\
&\quad - [a \leftrightarrow b, \alpha \leftrightarrow \beta] \Big\} \partial_\mu \,, \\
(T_4)^{ab\alpha\beta} &= \frac{h}{8} d^{cde} \Big[\Big(-4a\theta^{\alpha\rho} (A_\rho)^{ac} (A_\mu)^{bd} F^{e\beta\mu} - a\theta^{\rho\sigma} (A^\mu)^{ac} (A_\mu)^{bd} F^e_{\rho\sigma} g^{\alpha\beta} \\
&\quad + a\theta^{\rho\sigma} (A^\beta)^{ac} (A^\alpha)^{bd} F^e_{\rho\sigma} - 4\theta^{\alpha\rho} (A^\beta)^{ad} (A^\mu)^{bc} F^e_{\mu\rho} \\
&\quad + 4\theta^{\alpha}{}_{\rho} (A^\mu)^{ad} (A_\mu)^{bc} F^{e\beta\rho} + 4\theta^{\nu}{}_{\rho} (A_\nu)^{ad} (A_\mu)^{bc} F^{e\mu\rho} g^{\alpha\beta} \\
&\quad + 2\theta^{\alpha\beta} (A_\mu)^{ad} (A_\nu)^{bc} F^{e\mu\nu} + 4\theta^{\alpha\rho} (A_\mu)^{ad} (A_\rho)^{bc} F^{e\beta\mu} \\
&\quad + 2\theta^{\rho\sigma} (A_\rho)^{ad} (A_\sigma)^{bc} F^{e\alpha\beta} \Big) + (a \leftrightarrow b \,, \alpha \leftrightarrow \beta) \\
&\quad + f^{abc} (2a\theta^{\rho\sigma} F^d_{\rho\sigma} F^{e\alpha\beta} + a\theta^{\alpha\beta} F^d_{\rho\sigma} F^{e\rho\sigma} \\
&\quad + 4\theta_{\rho\sigma} F^{d\alpha\rho} F^{e\beta\sigma} + 8\theta^{\alpha\rho} F^{d\beta\mu} F^e_{\mu\rho}) \Big] \,.
\end{aligned}
$$

The divergent parts are calculated in the momentum representation via dimensional regularization

$$
\begin{aligned}
\mathrm{D_1}^{\mathrm{div}} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_1\right)^2\left(\Box^{-1}T_4\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x \left[\frac{a-3}{4}(\theta^{\alpha\rho}F^a_{\alpha\nu}+\theta_{\alpha\nu}F^{a\alpha\rho})(V^\nu V^\mu V_\mu V_\rho)^{bc}\right. \\
&\qquad\left. +\frac{3a-4}{4}\theta^{\rho\sigma}F^a_{\rho\sigma}(V^\mu V^\nu V_\nu V_\mu)^{bc}\right], \\
\mathrm{D_2}^{\mathrm{div}} &= -\frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_1\right)^3\left(\Box^{-1}T_3\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x \left[\frac{7-3a}{6}\theta^{\rho\sigma}F^a_{\rho\sigma}(V_\mu V^\mu V_\nu V^\nu + V_\mu V_\nu V^\mu V^\nu + V_\mu V_\nu V^\nu V^\mu)^{bc}\right. \\
&\qquad + \frac{3-2a}{6}(\theta^{\alpha\rho}F^a_{\alpha\sigma}+\theta_{\alpha\sigma}F^{a\alpha\rho})(V_\rho V^\sigma V^\mu V_\mu + V_\rho V_\mu V^\sigma V^\mu \\
&\qquad\left. + V_\rho V^\mu V_\mu V^\sigma)^{bc}\right], \\
\mathrm{D_3}^{\mathrm{div}} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_1\right)^4\left(\Box^{-1}T_2\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x \left[\frac{7a-11}{12}\theta^{\rho\sigma}F^a_{\rho\sigma}(V_\mu V^\mu V_\nu V^\nu + V_\mu V_\nu V^\mu V^\nu + V_\mu V_\nu V^\nu V^\mu)^{bc}\right. \\
&\qquad + \frac{a-3}{12}(\theta^{\alpha\rho}F^a_{\alpha\sigma}+\theta_{\alpha\sigma}F^{a\alpha\rho})(2V^\mu V_\mu V_\rho V^\sigma + 2V^\mu V_\rho V_\mu V^\sigma \\
&\qquad\left. + V^\mu V_\rho V^\sigma V_\mu + V_\rho V^\mu V_\mu V^\sigma)^{bc}\right],
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{D_4}^{\mathrm{div}} &= -\frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_2\right)\left(\Box^{-1}T_4\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc}\int \mathrm{d}^4x\, \Big[\frac{4-3a}{4}\theta^{\rho\sigma}F^a_{\rho\sigma}(V_\mu V_\nu V^\nu V^\mu)^{bc} \\
&\qquad + \frac{2-a}{2}(\theta^{\alpha\rho}F^a_{\alpha\mu} + \theta_{\alpha\mu}F^{a\alpha\rho})(V^\mu V^\nu V_\nu V_\rho)^{bc} \\
&\qquad + i(2(a+1)\theta_{\alpha\nu}F^a_{\beta\mu}(V^\mu F^{\alpha\beta}V^\nu)^{bc} + \theta^{\alpha\beta}F^a_{\mu\nu}(V^\mu F_{\alpha\beta}V^\nu)^{bc}) \\
&\qquad + 2i\theta^{\alpha\beta}F^a_{\beta\mu}\left(V_\nu F^{\mu\nu}V_\alpha - V^\mu F^{\alpha\beta}V^\nu\right)^{bc} \\
&\qquad - i\theta^{\alpha\beta}F^a_{\mu\nu}(V_\alpha F^{\mu\nu}V_\beta)^{bc} - 2N\theta^{\beta\mu}F^a_{\mu\nu}F^{b\alpha\nu}V^c_{\alpha\beta} \\
&\qquad + N\theta^{\mu\nu}F^a_{\alpha\mu}F^b_{\beta\nu}F^{c\alpha\beta} - a\frac{3}{4}N\theta^{\mu\nu}F^a_{\mu\nu}F^b_{\alpha\beta}F^{c\alpha\beta}\Big], \\
\mathrm{D_5}^{\mathrm{div}} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_2\right)^2\left(\Box^{-1}T_2\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x\, \Big[\frac{a-3}{2}(\theta_{\alpha\rho}F^{a\alpha\mu} + \theta^{\alpha\mu}F^a_{\alpha\rho})(F_{\mu\nu}F^{\nu\rho})^{bc} \\
&\qquad + \theta^{\rho\sigma}F^a_{\rho\sigma}\left(\frac{3a-4}{4}(F_{\mu\nu}F^{\mu\nu})^{bc} + \frac{3a-7}{4}(V^\mu V_\mu V^\nu V_\nu)^{bc}\right)\Big], \\
\mathrm{D_6}^{\mathrm{div}} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\left(\Big[\left(\Box^{-1}N_1\right)\left(\Box^{-1}N_2\right) + \left(\Box^{-1}N_2\right)\left(\Box^{-1}N_1\right)\Big]\left(\Box^{-1}T_3\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x\, \Big[\frac{a-3}{2}\theta^{\alpha\sigma}F^a_{\rho\sigma}(V_\alpha V_\beta V^\beta V^\rho)^{bc} - \frac{a}{4}\theta^{\rho\sigma}F^a_{\rho\sigma}(F_{\alpha\beta}F^{\alpha\beta})^{bc} \\
&\quad - \theta_{\alpha\sigma}F^a_{\beta\rho}(a+1)\left(2\mathrm{i}(V^\alpha F^{\rho\sigma}V^\beta)^{bc} - (F^{\rho\sigma}F^{\alpha\beta})^{bc}\right) \\
&\quad + \frac{a}{2}\theta^{\beta\sigma}F^a_{\rho\sigma}(V_\alpha V^\alpha V_\beta V^\rho + V_\alpha V^\alpha V^\rho V_\beta)^{bc} \\
&\quad + \theta^{\rho\sigma}F^a_{\rho\sigma}(\frac{3a-4}{4}V_\alpha V_\beta V^\beta V^\alpha + \frac{5a-4}{4}V_\alpha V^\alpha V_\beta V^\beta - 2V_\alpha V_\beta V^\alpha V^\beta)^{bc}\Big] + \ldots,
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{D}_7{}^{\mathrm{div}} &= -\frac{\mathrm{i}}{2}\mathrm{Tr}\left(\sum\left(\Box^{-1}N_1\right)^2\left(\Box^{-1}N_2\right)\left(\Box^{-1}T_2\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x \left[\frac{18-11a}{12}\theta^{\alpha\beta}F^a_{\alpha\beta}(2V^\mu V_\mu V^\nu V_\nu + V^\mu V^\nu V_\nu V_\mu)^{bc}\right. \\
&+ \left.\frac{3-a}{6}(\theta^{\alpha\mu}F^a_{\beta\mu}+\theta_{\beta\mu}F^{a\alpha\mu})(V_\alpha V^\beta V^\nu V_\nu + V^\nu V_\nu V^\beta V_\alpha + V_\alpha V^\nu V_\nu V^\beta)^{bc}\right].
\end{aligned}
$$

$$
\sum_{i=1}^{7}\mathsf{D}_i{}^{\mathrm{div}} = \frac{N}{(4\pi)^2\epsilon}h\theta^{\mu\nu}d^{abc}\int \mathrm{d}^4x\left(-\frac{25a-3}{48}F^a_{\mu\nu}F^b_{\rho\sigma}+\frac{a+21}{12}F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma},
$$

Renormalization of the theory:

To cancel divergences, counter terms should be added to the starting action, which produces the bare Lagrangian

$$
\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{11Ng^2}{6(4\pi)^2\epsilon}F^a_{\mu\nu}F^{a\mu\nu} \\
&+ \frac{1}{4}g\mu^{\epsilon/2}h\theta^{\mu\nu}d^{abc}\left(\frac{a}{4}F^a_{\mu\nu}F^b_{\rho\sigma} - F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma} \\
-&\frac{Ng^3\mu^{\epsilon/2}}{(4\pi)^2\epsilon}h\theta^{\mu\nu}d^{abc}\left(\frac{3-25a}{48}F^a_{\mu\nu}F^b_{\rho\sigma}+\frac{21+a}{12}F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma} \\
&= -\frac{1}{4}F_0{}^a_{\mu\nu}F_0{}^{a\mu\nu} + \frac{1}{4}g\mu^{\epsilon/2}h\theta^{\mu\nu}d^{abc} \\
&\times \left[\frac{a}{4}\left(1-\frac{3-25a}{3a}\frac{Ng^2}{(4\pi)^2\epsilon}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\right. \\
&\quad \left. -\left(1+\frac{21+a}{3}\frac{Ng^2}{(4\pi)^2\epsilon}\right)F^a_{\mu\rho}F^b_{\nu\sigma}\right]F^{c\rho\sigma}.
\end{aligned}
$$

To obtain the same structure as in starting Lagrangian we have to impose the condition

$$\Big(-\frac{25a-3}{48}\Big) : \Big(\frac{a+21}{12}\Big) = \frac{a}{4} : (-1).$$

Solutions, $a = 1$ and $a = 3$.

The case $a = 1$ correspondes to Model 1 : mNCSM; the deformation parameter h need not to be renormalized. Renormalization obtained through the renormalizations of gauge fields the coupling constant.

The case $a = 3$ is different since the NC deformation parameter h has to be renormalized.

The bare gauge field, the coupling constant and the NC deformation parameter are :

$$\begin{aligned} V_0^\mu &= V^\mu \sqrt{1 + \frac{22Ng^2}{3(4\pi)^2\epsilon}}, \\ g_0 &= \frac{g\mu^{\epsilon/2}}{\sqrt{1 + \frac{22Ng^2}{3(4\pi)^2\epsilon}}}, \\ h_0 &= \frac{h}{1 - \frac{2Ng^2}{3(4\pi)^2\epsilon}}, \end{aligned}$$

Ultraviolet asymptotic behaviour of NC SU(N) GFT via RGE

Gauge coupling constant g in our theory depends on energy i.e., the renormalization point μ, *satisfying the same beta function* as in QCD

$$\beta_g = \mu \frac{\partial}{\partial \mu} g(\mu) = -\frac{11 N g^3(\mu)}{3(4\pi)^2},$$

our theory is UV *stable, i.e.* $asymptotically\ free$:

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi} = \frac{6\pi}{11N} \frac{1}{\ln \frac{\mu}{\Lambda}}.$$

Λ is an integration constant not predicted by the theory; free parameter to be determined from experiment: hadronic production in e^+e^- annihilation at the Z resonance has given $\alpha_s(m_Z) = 0.12$ corresponding to $\Lambda = \Lambda_{\text{QCD}} \simeq 250$ MeV.

$$\beta_h = \mu \frac{\partial}{\partial \mu} h(\mu) = -\frac{11 N g^2(\mu)}{24\pi^2} h(\mu).$$

Both β functions are *negative* $\rightarrow$ decrese with increasing energy μ. Solution to β_h:

$$h(\mu) = \frac{h_0}{\ln \frac{\mu}{\Lambda}}, \quad \Rightarrow$$

$running\ deformation\ parameter\ h$. h_0 is an additional integration constant, physical interpretation later.

By increase of μ the h decreases, $\Rightarrow$ modification of Heisenberg uncertainty relations at high energy

$$\begin{aligned} [x,p] &= i\hbar(1+\beta p^2), \\ \Delta x &= \frac{\hbar}{2}(\frac{1}{\Delta p}+\beta\Delta p). \end{aligned}$$

Large momenta $\rightarrow$ distance Δx grows linearly: So large energies do not necessarily correspond to small distances. Running h does not imply that noncommutativity vanishes at small distances. Related to UV/IR correspondence. By assuming

$$h(\mu)=\frac{1}{\Lambda^2_{\rm NC}(\mu)}$$

$\Lambda_{\rm NC}$ becomes a function of energy μ too giving:

$$\begin{aligned} \mu\frac{d}{d\mu}\Lambda_{\rm NC}(\mu) &= \frac{11N}{3(4\pi^2)}g^2(\mu)\,\Lambda_{\rm NC}(\mu)\,, \\ \Lambda_{\rm NC}(\mu) &= \Lambda_\theta\sqrt{\ln\frac{\mu}{\Lambda}}. \end{aligned}$$

$\Lambda_{\rm NC}$ becomes the running scale of non-commutativity.
* Physical interpretation of h_0 and/or Λ_θ is not quite clear; they have to be proportional to the scale of noncommutativity $\Lambda_{\rm NC}$.
* Assume that in a first approximation $h_0=1/\Lambda^2_\theta=1/\Lambda^2_{\rm NC}$.
* Considering typical QCD energies, $\mu=m_Z$, factor $\sqrt{\ln(m_Z/\Lambda_{\rm QCD})}\simeq 2.4$

FORBIDDEN DECAYS

GAUGE SECTOR: $Z \to \gamma\gamma$ decay

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\to \gamma\gamma$, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; JHEP**03** (2007) 030]

[M. Buric, D. Latas, V. Radovanovic and J.T., Nonzero Z $\to \gamma\gamma$ decay in the renormalizable NCSM; Phys. Rev. **D 75**, 097701 (2007).]

From $\mathcal{L}_{Z\gamma\gamma} \Rightarrow$ the gauge-invariant amplitude $\mathcal{A}_{Z\to\gamma\gamma}$

$$\begin{aligned} \mathcal{A}^\theta(Z \to \gamma\gamma) &= -2e \sin 2\theta_W \, \mathrm{K}_{Z\gamma\gamma} \Theta_3^{\mu\nu\rho}(a; k_1, -k_2, -k_3) \\ &\times \epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(k_3); \end{aligned}$$

$k_1 + k_2 + k_3 = 0$;

$$\begin{aligned} \Theta_3^{\mu\nu\rho}(a; k_1, k_2, k_3) &= -(k_1\theta k_2) \\ &\times[(k_1 - k_2)^\rho g^{\mu\nu} + (k_2 - k_3)^\mu g^{\nu\rho} + (k_3 - k_1)^\nu g^{\rho\mu}] \\ &- \theta^{\mu\nu}\,[k_1^\rho\,(k_2k_3) - k_2^\rho\,(k_1k_3)] \\ &- \theta^{\nu\rho}\,[k_2^\mu\,(k_3k_1) - k_3^\mu\,(k_2k_1)] \\ &- \theta^{\rho\mu}\,[k_3^\nu\,(k_1k_2) - k_1^\nu\,(k_3k_2)] \\ &+ (\theta k_2)^\mu\,\left[g^{\nu\rho}\,k_3^2 - k_3^\nu k_3^\rho\right] + (\theta k_3)^\mu\,\left[g^{\nu\rho}\,k_2^2 - k_2^\nu k_2^\rho\right] \\ &+ (\theta k_3)^\nu\,\left[g^{\mu\rho}\,k_1^2 - k_1^\mu k_1^\rho\right] + (\theta k_1)^\nu\,\left[g^{\mu\rho}\,k_3^2 - k_3^\mu k_3^\rho\right] \\ &+ (\theta k_1)^\rho\,\left[g^{\mu\nu}\,k_2^2 - k_2^\mu k_2^\nu\right] + (\theta k_2)^\rho\,\left[g^{\mu\nu}\,k_1^2 - k_1^\mu k_1^\nu\right] \\ &+ \theta^{\mu\alpha}(ak_1 + k_2 + k_3)_\alpha\,\left[g^{\nu\rho}\,(k_3k_2) - k_3^\nu k_2^\rho\right] \\ &+ \theta^{\nu\alpha}(k_1 + ak_2 + k_3)_\alpha\,\left[g^{\mu\rho}\,(k_3k_1) - k_3^\mu k_1^\rho\right] \\ &+ \theta^{\rho\alpha}(k_1 + k_2 + ak_3)_\alpha\,\left[g^{\mu\nu}\,(k_2k_1) - k_2^\mu k_1^\nu\right]\,. \end{aligned}$$

Experiments

Decay mode: $Z \to \gamma\gamma$, old mesurements:

$$BR = \frac{\Gamma(Z \to \gamma\gamma)}{\Gamma_{tot}(Z)} \begin{cases} < 5.2 \times 10^{-5} & \text{L3} & 1995 \\ < 5.5 \times 10^{-5} & \text{DELPHI} & 1994 \\ < 1.4 \times 10^{-4} & \text{OPAL} & 1991 \end{cases}$$

$e^+e^- \to \gamma\gamma$ near Z resonance is an ideal process to test QED. The present statistic enables comparison of data with the QED up to $\mathcal{O}(\alpha^3)$.

Deviation of the experimentaly measured cross sections from the QED prediction $\longrightarrow$ evidence for $Z \to \gamma\gamma$ (SM forbidden) and $Z \to \pi^0\gamma \,/\, \eta\gamma$.

The forbidden decay $Z \to \gamma\gamma$ and the real decays $Z \to \pi^0\gamma \,/\, \eta\gamma$ would have the same experimental signature as the SM forbidden process

$$e^+e^- \to Z^* \to \gamma\gamma$$

Rare decays at high energies, the two photons from π^0 or η decays are very close seen in EM calorimeter as a single high energy photon:

$$e^+e^- \to Z^* \to (\pi^0, \eta)\gamma \to (\gamma\gamma)\gamma$$

Theoretical estimates $Br(Z \to \pi^0\gamma \,/\, \eta\gamma) \sim 10^{-10}$.
(Arnellos at al. Nucl.Phys.B 196 (1982) 378)

$Z \to \gamma\gamma$ LHC experimental possibilities:

CMS Physics Technical Design Report:

10^7 events of $Z \to e^+e^-$ for 10 fb^{-1} in 2 years of LHC

Assuming $BR(Z \to \gamma\gamma) \sim 10^{-8}$ and using $BR(Z \to e^+e^-) = 0.03 \Rightarrow \sim 3$ events of $Z \to \gamma\gamma$ with 10 fb^{-1}

Background sources (CMS Note 2006/112, Fig.3):
1. Study for $Higgs \to \gamma\gamma$ showes that, when e^- from $Z \to e^+e^-$ radiates very high energy Bremsstrahlung photon into pixel detector, for similar energies of e^- and γ, there is a huge probability of misidentification of e^- with γ !
2. Irreducible di-photon background may kill signal.

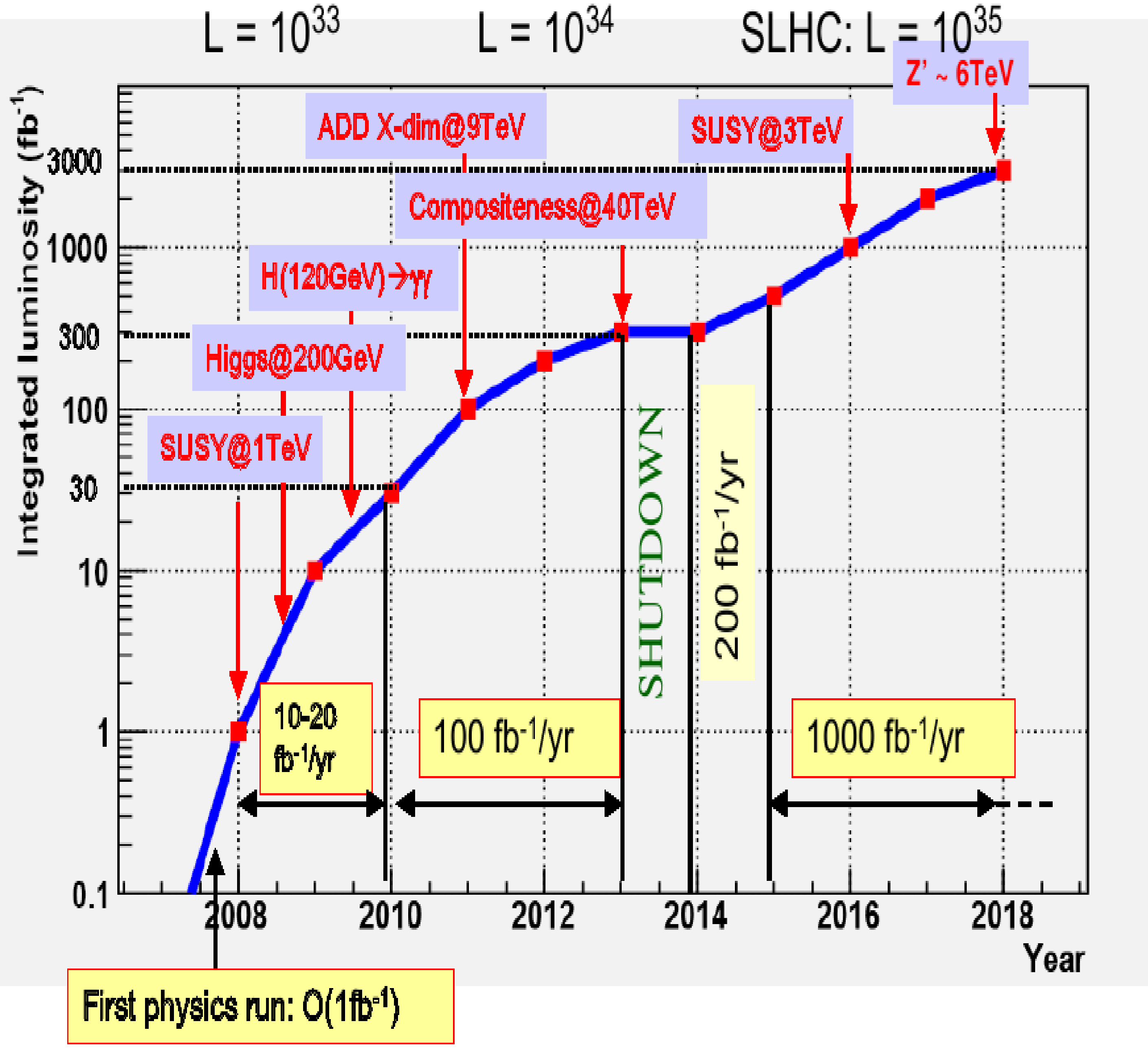
After 10 years of LHC running Int. L $\sim$ 1000 fb^{-1} and assuming $BR(Z \to \gamma\gamma) \sim 10^{-8}$

$\Rightarrow \sim 300$ events of $Z \to \gamma\gamma$ decays, OR

$\Rightarrow \sim 3$ events with $BR(Z \to \gamma\gamma) \sim 10^{-10}$

$\Rightarrow$ NC scale $\Lambda_{NC} \gtrsim 3.0$ TeV

Probable/possible LHC luminosity profile - need for L-upgrade in a longer term



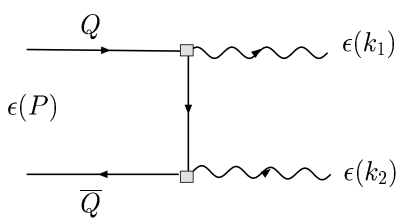
for the 2008 run likely to get from 100pb^{-1} to 1fb^{-1}

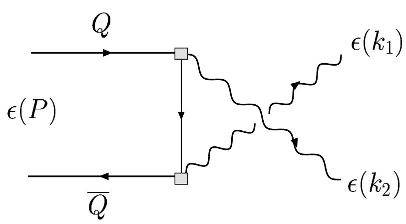
HADRON SECTOR

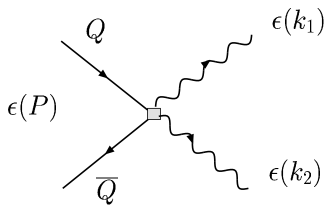
* NEUTRAL CURRENT DECAYS:

$\overline{Q}Q_{1^{--}}(J/\psi,\, \Upsilon) \to \gamma\gamma$

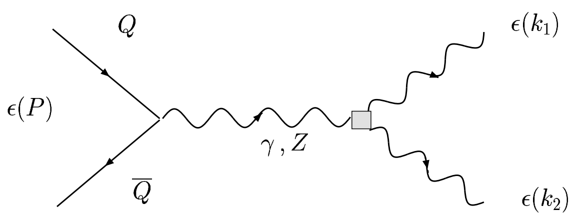
[B. Melic, K. Passek-Kumericki and J.T.; Quarkonia decays into two photons induced by the space-time noncommutativity, PRD **72** (2005) 054004]







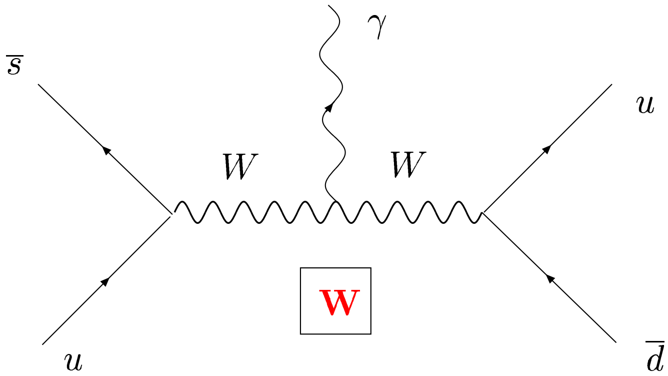
$$\mathcal{M}_{\text{mNCSM}} = i\,\pi\,4\sqrt{3\,M}\,\alpha\, e_Q^2 |\Psi_{\overline{Q}Q}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P)$$
$$\times \Big\{ -(k_1 - k_2)^\rho \left[\theta^{\mu\nu} - 2g^{\mu\nu}\frac{(k_1\theta k_2)}{M^2}\right]$$
$$+2g^{\mu\rho}\left[(k_1\theta)^\nu - 2k_1^\nu \frac{(k_1\theta k_2)}{M^2}\right] + 2g^{\nu\rho}\left[(k_2\theta)^\mu + 2k_2^\mu \frac{(k_1\theta k_2)}{M^2}\right] \Big\}$$



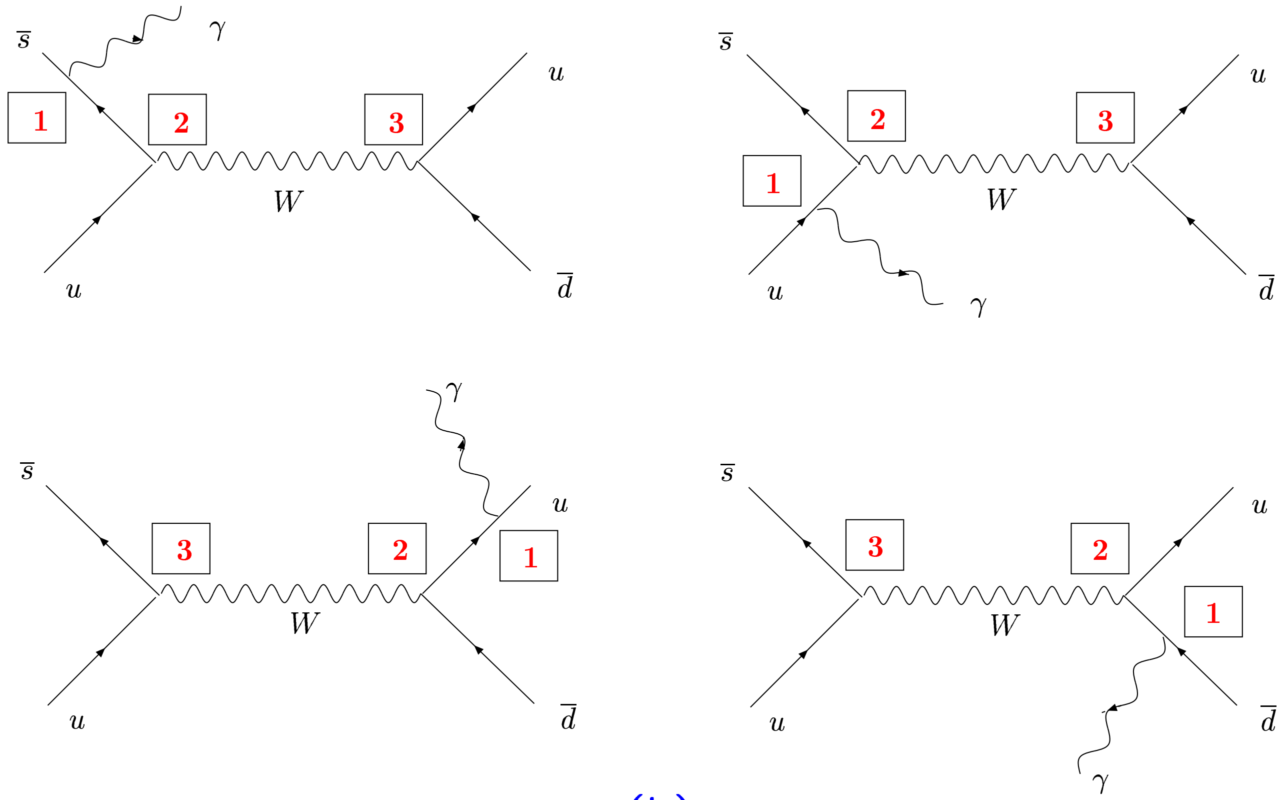
$$\mathcal{M}_{\text{nmNCSM}} = -\,i\,\pi\frac{16\sqrt{3\,M}}{M^2}\alpha|\Psi_{\overline{Q}Q}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P)$$
$$\times\Theta_3((\mu,k_1),(\nu,k_2),(\rho,P))\left[e_Q \sin 2\theta_W K_{\gamma\gamma\gamma} + \left(\frac{M}{M_Z}\right)^2 c_V^Q K_{Z\gamma\gamma}\right]$$

* CHARGED CURRENT DECAYS: $K \to \pi\gamma$,...

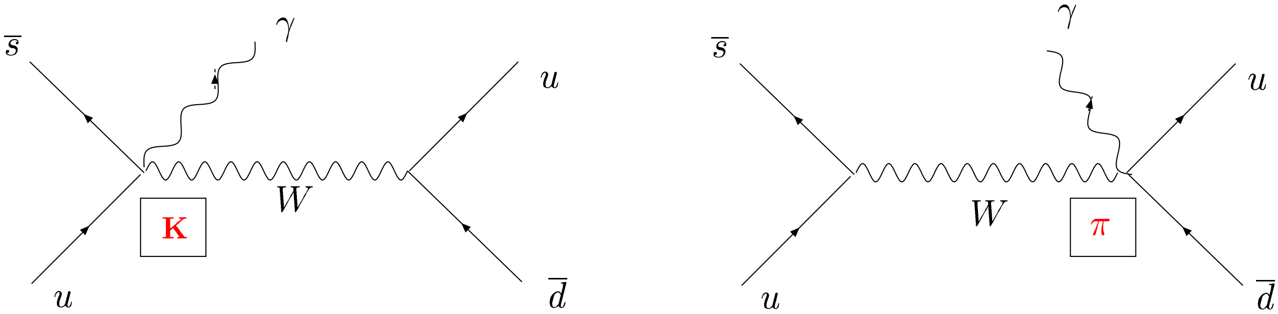
[B. Melic, K. Passek-Kumericki and J.T.; K → pi gamma decay and space-time noncommutativity, Phys. Rev. D **72** (2005) 057502]



(a)



(b)



(c)

DISCUSION

Limits on Λ_{NC} from theory and experiment

DECAYS: 1→ 2

* $Z \to \gamma\gamma \quad \Rightarrow \Lambda_{NC} > \binom{110}{1000}$ GeV, [Duplančić,...];[Burić,...]

* $\gamma_{pl} \to \nu\bar{\nu} \quad \Rightarrow \Lambda_{NC} >$ 81 GeV, [Schupp, JT, Wess, Raffelt]

* $J/\psi \to \gamma\gamma \quad \Rightarrow \Lambda_{NC} >$ 9 GeV, [Melic, Passek, J.T.]

* $K \to \pi\gamma \quad \Rightarrow \Lambda_{NC} >$ 43 GeV, [Melic, Passek, J.T.]

SCATTERINGS: 2→ 2

* $e^+e^- \to \gamma\gamma \Rightarrow \Lambda_{NC} >$ 141 GeV, [OPAL Coll. (2003)]

* $\gamma\gamma \to \bar{f}f \quad \Rightarrow \Lambda_{NC} >$ 200 GeV, [T. Ohl et al.]

* $\bar{f}f \to Z\gamma \quad \Rightarrow \Lambda_{NC} >$ 1 TeV, [T. Ohl et al.]

SUMMARY

⋆ Principle of renormalizability implemented on our θ-expanded NCGFT led us to well defined deformations via introduction of higher order NC gauge action class for mNCSM, nmNCSM and NC SU(N) models. This extension was parametrized by generically free parameter a.

⋆ Divergences cancel differently than in commutative GFT and this depends on the representations.

⋆ Model 1: mNCSM gauge sector is renormalizable for $a = 1$. No renormalizaion of h.

⋆ Model 2: nmNCSM gauge sector is renormalizable and FINITE for $a = 3$. No renormalizaion of h.

⋆ Model 3: NC SU(N) theory is renormalizable only for $a = 1, 3$.
− Case $a = 1$, requires no renormalizaion of h.
− Case $a = 3$, requires renormalization of NC deformation parameter h, which becomes *the running deformation parameter and vanishes for large* μ.
− Λ_{NC} runs too and it is very smooth, ⇒ small change when μ increases ⇒ large degree of stability of NC SU(N) theory within a wide range of μ.

CONCLUSION

$\star$ Renormalization principle is fixing the freedom parameter $a = 1, 3$ for our θ-expanded NC GFT :

$$S^a_\star = -\frac{1}{2}\text{Tr}\int d^4x \left(1 + i(a-1)\,\widehat{x}^\mu \star \widehat{x}^\nu \star \widehat{F}_{\mu\nu}\right) \star \widehat{F}_{\rho\sigma} \star \widehat{F}^{\rho\sigma}.$$

This way principle of renormalization determines NC renormalizable deformation.

$\star$ The solution $a = 3$, while shifting the model to the higher order, hints into the discovery of the key role of the higher NC gauge interaction in 1-loop renormalizability of classes of NCGFT at θ^1.

$\star$ Hence, the nmNCSM gauge sector, which produces SM forbidden $Z \to \gamma\gamma$ decay, is renormalizable and FINITE $\to$ no renormalizaion of h needed.

$\star$ Hence, in the case of NC SU(N) the noncommutativity deformation parameter h had to be renormalized and it is asymptotically free, opposite to the previous expectations.

$\star$ Similarity to ϕ^4 NC GFT: Adding $\Omega \int d^4x\, \widehat{x}\star\widehat{x}\star\widehat{\phi}\star\widehat{\phi}$. renormalization principle determines NC renormalizable deformation up to all orders.

$\star$ We beleive that all above could be of paramounth importance in modifying matter sector so that it becomes renormalizable.

$\star$ Phenomenological results as $Z \to \gamma\gamma$ are ROBUST due to the 1-loop renormalizability and finitness of the nmNCSM gauge sector.